第四章 CMOS单元电路

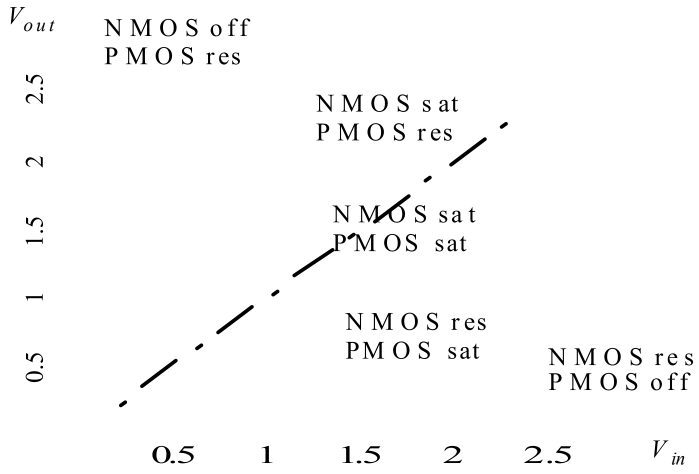
4.2 反相器瞬态特性

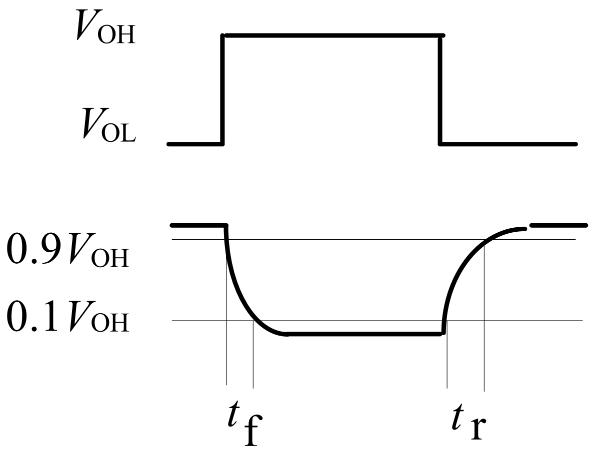
CMOS反相器

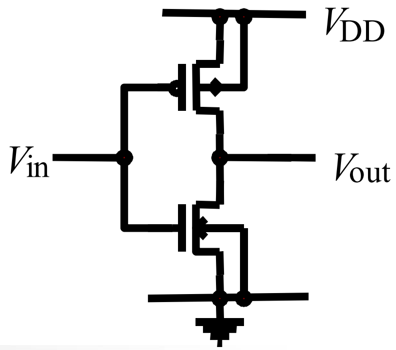
- 4.1 CMOS反相器的直流特性
- 4.2 CMOS反相器的瞬态特性
- 4.3 CMOS反相器的设计

直流特性和瞬态特性

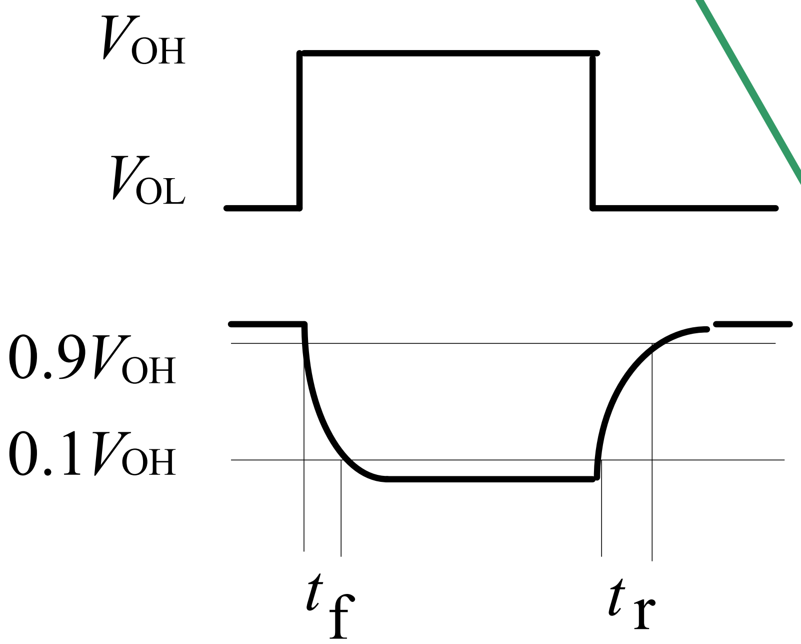
- 直流特性有助于我们理解反相器中器件的工作状态和电路的噪声特性
- 瞬态特性，即输入信号随着时间变化过程中，输出信号的变化情况
- 瞬态特性决定着电路的速度







1、上升时间和下降时间



(1) 出现上升/下降的原因**:**
Vin跳变**(**由**0**到**1**，或相反**)**，
Vout不会立刻反相

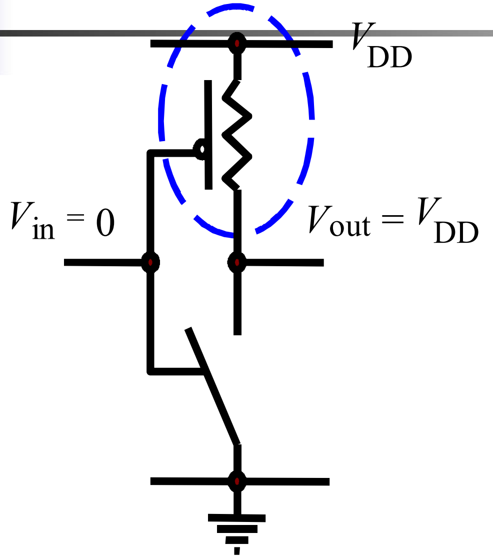
(2) **Issue**：**Vout**不会立刻反相
的原因？

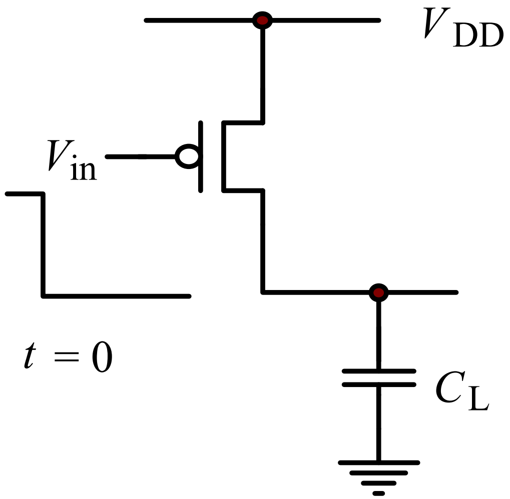
(3) 上升时间 **rise-time/**下降时
间 **fall-time (tr/tf)**的定义

上升时间**:** 输出从逻辑摆幅的**10%**
10%变化到**90%**

下降时间**:** 输出从逻辑摆幅的**90%**
90%变化到**10%**

分析上升时间的等效电路





(1) 物理思想：

通过**PMOS**对**Vout**节点的电容充电

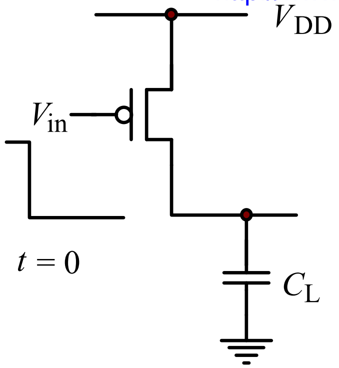
$$C_L \frac{dV_{out}}{dt} = I_{DP}$$

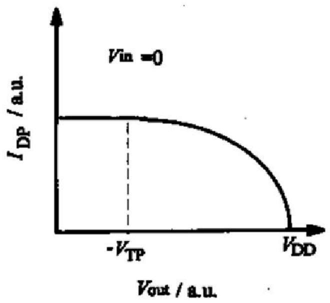
(2) I_{DP}是随输出变化的

$V_{out} <= |V_{TP}|$, PMOS在饱和区，

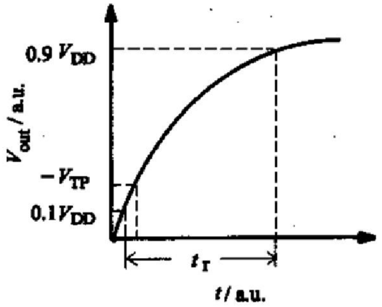
$V_{out} > |V_{TP}|$, PMOS在线性区

推导上升时间





(a) 充电电流的变化



(b) 输出电平的变化

1) $V_{out} <= -V_{TP}$ **PMOS饱和**

$$C_L \frac{dV_{out}}{dt} = K_P(0 - V_{TP} - V_{DD})^2$$

归一化

$$\frac{du}{dt} = \frac{1}{\tau_r}(1-\alpha_P)^2, \qquad \tau_r = \frac{C_L}{K_P V_{DD}}$$

积分求解

$$t_1 = \frac{\tau_r(\alpha_P - u_1)}{(1-\alpha_P)^2}$$

2) $V_{out} > -V_{TP}$ **PMOS线性**

$$\frac{du}{dt} = \frac{1}{\tau_r}\left[(1-\alpha_P)^2 - (u-\alpha_P)^2\right]$$

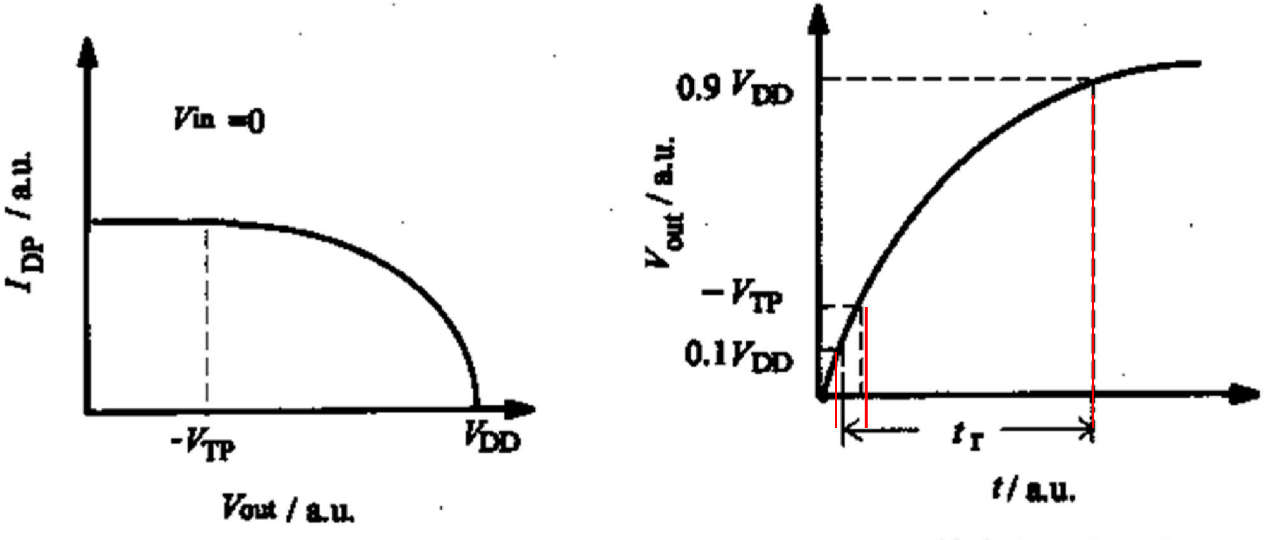
积分求解

$$t_2 = \frac{\tau_r}{2(1-\alpha_P)}\ln\left(\frac{1+u_2-2\alpha_P}{1-u_2}\right)$$

上升时间

$$t_r = t_1(u_1 = 0.1) + t_2(u_2 = 0.9)$$

上升过程充电电流的变化



(a) 充电电流的变化 (b) 输出电平的变化

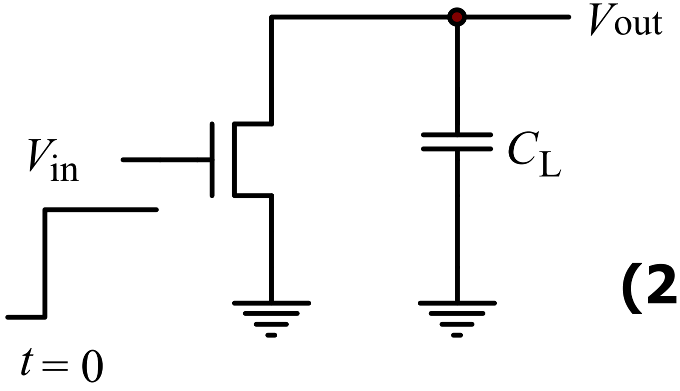
$$t_r = \tau_r\left[\frac{\alpha_P - 0.1}{(1-\alpha_P)^2} + \frac{1}{2(1-\alpha_P)}\ln\left(\frac{1.9-2\alpha_P}{0.1}\right)\right]$$

$$\tau_r = \frac{C_L}{K_P V_{DD}}$$

Issue:
公式适用范围

$$0.1 \le \alpha_P \le 0.9$$

分析下降时间的等效电路



(1) 与上升电路类似的分析：
通过**NMOS**对**Vout**节点的
电容放电

$$C_L \frac{dV_{out}}{dt} = -I_{DN}$$

(2) Issue： I_{DN}的计算？

$\mathbf{Vout \geq V_{DD}-V_{TN}}$ **NMOS**饱和；

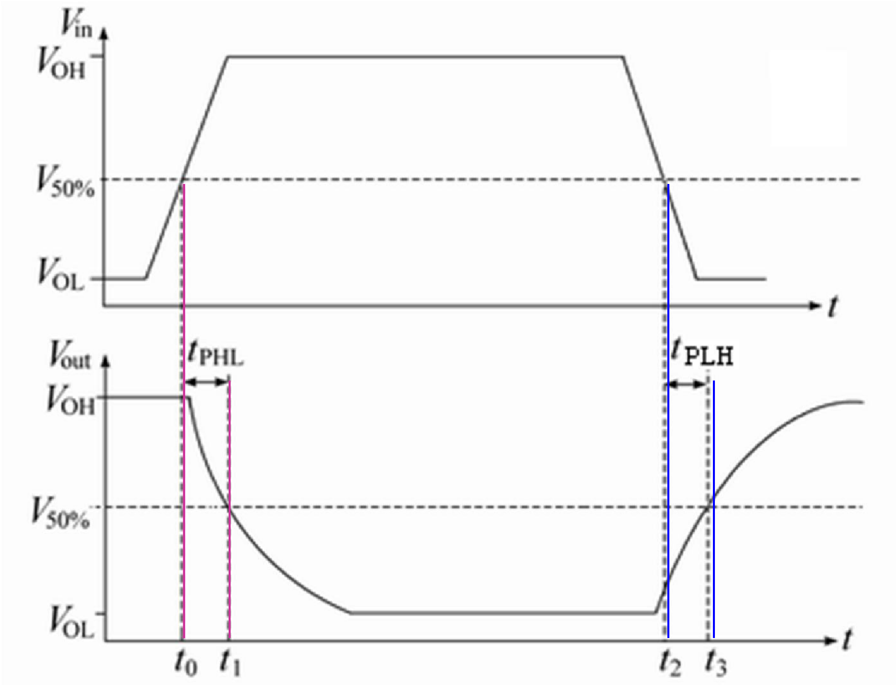
$\mathbf{Vout < V_{DD}-V_{TN}}$ **NMOS**线性

$$\left\{ \; t_f = \tau_f \left[\frac{\alpha_N - 0.1}{(1-\alpha_N)^2} + \frac{1}{2(1-\alpha_N)} \ln\left(\frac{1.9 - 2\alpha_N}{0.1} \right) \right], \quad \tau_f = \frac{C_L}{K_N V_{DD}} \right.$$

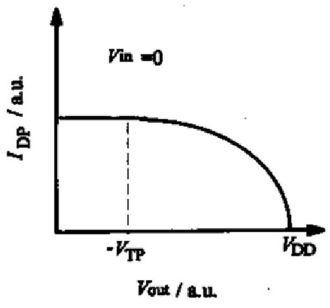
$$0.1 \leq \alpha_N \leq 0.9$$

2、反相器的传输延迟时间

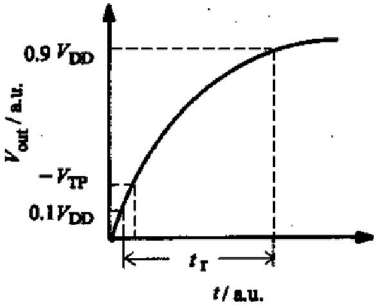
- 电路的工作速度取决于传输延迟时间
- 输入信号变化50％到输出信号变化50％的时间
- 根据输出信号情况，分为上升延迟和下降延迟时间



阶跃输入:上升延迟



(a) 充电电流的变化



(b) 输出电平的变化

1) $V_{\mathrm{out}} <= -V_{\mathrm{TP}}$ **PMOS饱和** $$C_{\mathrm{L}} \frac{\mathrm{d}V_{\mathrm{out}}}{\mathrm{d}t} = K_{\mathrm{P}}\left(0 - V_{\mathrm{TP}} - V_{\mathrm{DD}}\right)^2$$

归一化 $$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{1}{\tau_{\mathrm{r}}}\left(1 - \alpha_{\mathrm{P}}\right)^2, \qquad \tau_{\mathrm{r}} = \frac{C_{\mathrm{L}}}{K_{\mathrm{P}} V_{\mathrm{DD}}}$$

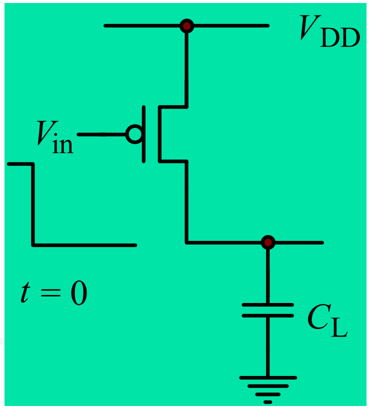
积分求解 $$t_1 = \frac{\tau_{\mathrm{r}}\left(\alpha_{\mathrm{P}} - u_1\right)}{\left(1 - \alpha_{\mathrm{P}}\right)^2}$$

2) $V_{\mathrm{out}} > -V_{\mathrm{TP}}$ **PMOS线性** $$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{1}{\tau_{\mathrm{r}}}\left[\left(1 - \alpha_{\mathrm{P}}\right)^2 - \left(u - \alpha_{\mathrm{P}}\right)^2\right]$$

积分求解 $$t_2 = \frac{\tau_{\mathrm{r}}}{2\left(1 - \alpha_{\mathrm{P}}\right)} \ln\left(\frac{1 + u_2 - 2\alpha_{\mathrm{P}}}{1 - u_2}\right)$$

上升延迟时间 $$t_{\mathrm{r}} = t_1\left(u_1 = 0\right) + t_2\left(u_2 = 0.5\right)$$

传输延迟时间：阶跃输入



- 输入信号变化到输出信号变化**50**%的时间

$$t_1=\frac{\tau_r\left(\alpha_P-u_1\right)}{\left(1-\alpha_P\right)^2}\qquad t_2=\frac{\tau_r}{2\left(1-\alpha_P\right)}\ln\left(\frac{1+u_2-2\alpha_P}{1-u_2}\right)$$

取：$t_r=t_1(u_1=0)+t_2(u_2=0.5)$

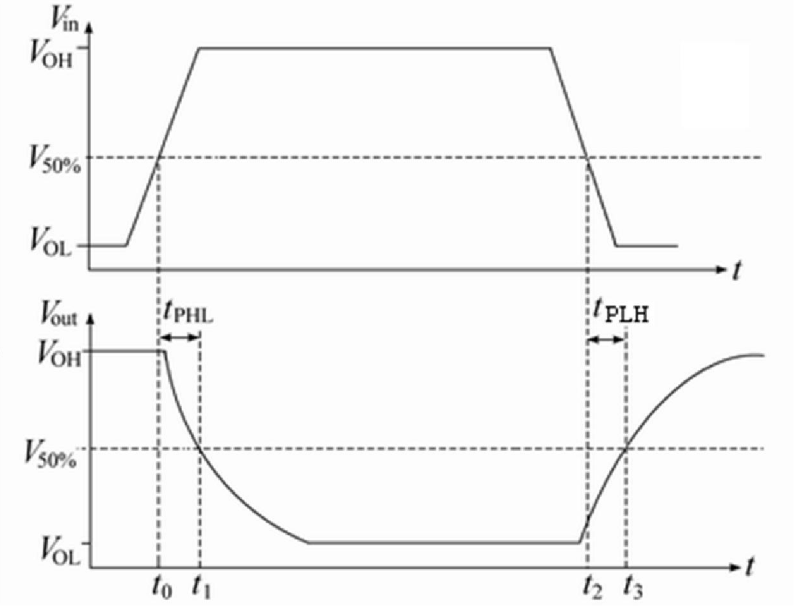
$$t_{pLH}=\frac{C_L}{K_p(V_{DD}-|V_{TP}|)}[\frac{|V_{TP}|}{V_{DD}-|V_{TP}|}+\frac{1}{2}\ln(\frac{4(V_{DD}-|V_{TP}|)}{V_{DD}}-1)]$$

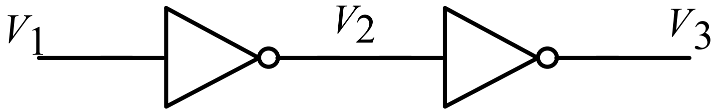
$$=\tau_r[\frac{\alpha_P}{(1-\alpha_P)^2}+\frac{1}{2(1-\alpha_P)}\ln(3-4\alpha_P)]$$

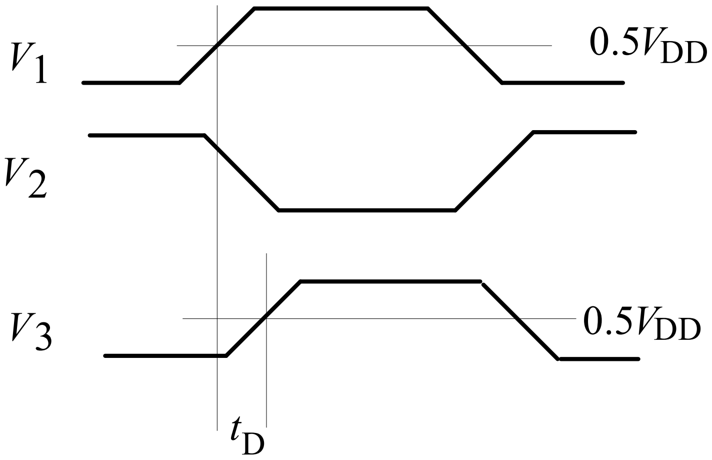
反相器的速度

- 用平均延迟时间表示一般情况下的速度

$$t_p = \frac{t_{pHL} + t_{pLH}}{2}$$







传输延迟：非阶跃输入近似

- 利用电流传送电荷的时间
- 电压变化**VDD/2**
- 电流取饱和电流的一半
- 精确的结果可以利用**SPICE**仿真

$$t_p = \frac{t_{pHL} + t_{pLH}}{2}$$

$$t_{pHL} = \frac{C_L \Delta V_{HL}}{I_{av,HL}} \approx \tau_f \frac{1}{\left(1-\alpha_N\right)^2}$$

$$t_{pLH} = \frac{C_L \Delta V_{LH}}{I_{av,LH}} \approx \tau_r \frac{1}{\left(1-\alpha_P\right)^2}$$

$$t_p = \frac{1}{2}\tau_r \left[\frac{1}{K_r\left(1-\alpha_N\right)^2} + \frac{1}{\left(1-\alpha_P\right)^2} \right]$$

延迟时间：影响因素

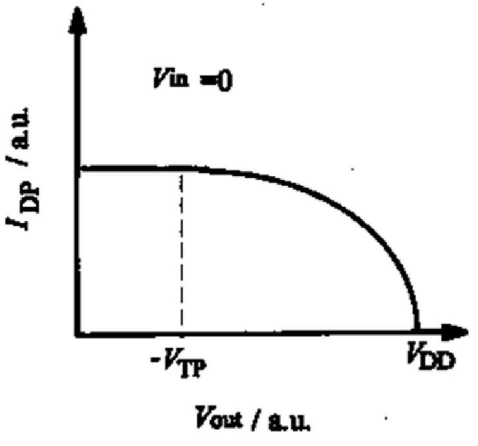
$$t_{\mathrm{p}} = \frac{t_{\mathrm{pHL}} + t_{\mathrm{pLH}}}{2}$$

$$t_{\mathrm{pHL}} = \frac{C_{\mathrm{L}} \Delta V_{\mathrm{HL}}}{I_{\mathrm{av,HL}}} \approx \tau_{\mathrm{f}} \frac{1}{\left(1-\alpha_{\mathrm{N}}\right)^{2}}$$

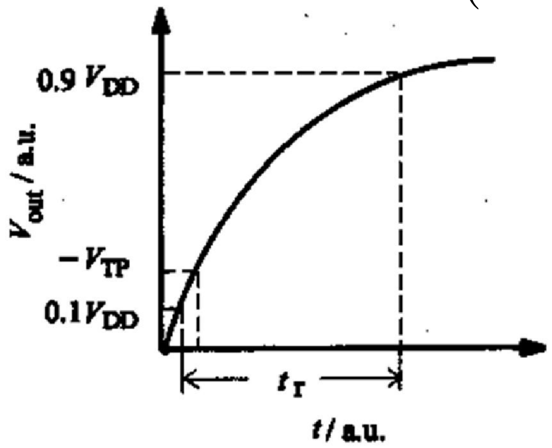
$$t_{\mathrm{pLH}} = \frac{C_{\mathrm{L}} \Delta V_{\mathrm{LH}}}{I_{\mathrm{av,LH}}} \approx \tau_{\mathrm{r}} \frac{1}{\left(1-\alpha_{\mathrm{P}}\right)^{2}}$$

➢上升和下降时间同电路充放电的电流和电容有关

➢因此，同器件的阈值电压，导电因子和电路的电源电压和负载电容有关



(a) 充电电流的变化

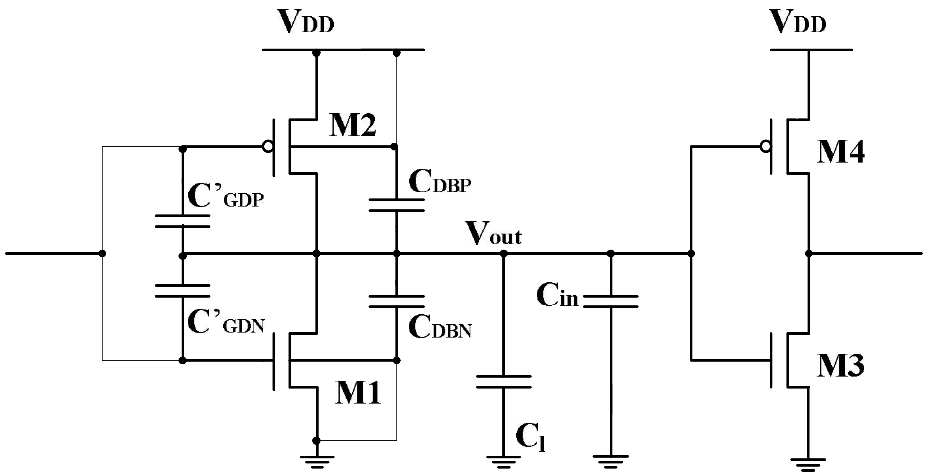


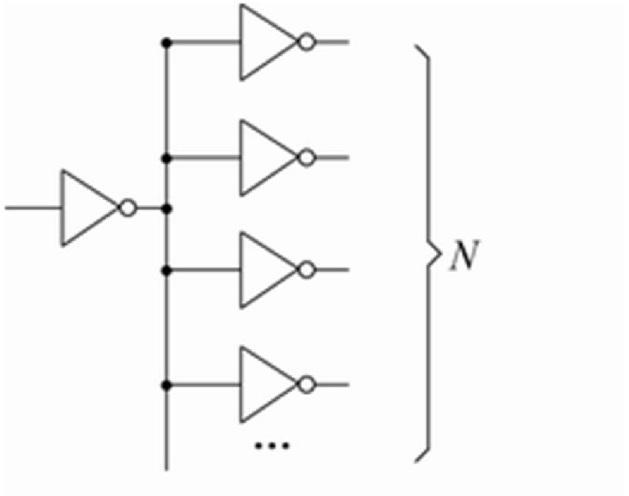
(b) 输出电平的变化

$$t_{\mathrm{p}} = \frac{1}{2} \tau_{\mathrm{r}} \left[\frac{1}{K_{\mathrm{r}} \left(1-\alpha_{\mathrm{N}}\right)^{2}} + \frac{1}{\left(1-\alpha_{\mathrm{P}}\right)^{2}} \right]$$

$$\tau_r = \frac{C_L}{K_P V_{DD}}$$

CMOS反相器的负载电容





(a) 扇出N

$$C_{\mathrm{L}} = C_{\mathrm{DBN}} + C_{\mathrm{DBP}} + \sum_{i=1}^{N} \left(W_{\mathrm{N}} + W_{\mathrm{P}}\right)_i L C_{\mathrm{ox}} + C_l$$

提高反相器的速度

$$C_{\mathrm{L}} = C_{\mathrm{DBN}} + C_{\mathrm{DBP}} + \sum_{i=1}^{N} \left(W_{\mathrm{N}} + W_{\mathrm{P}}\right)_i L C_{\mathrm{ox}} + C_l$$

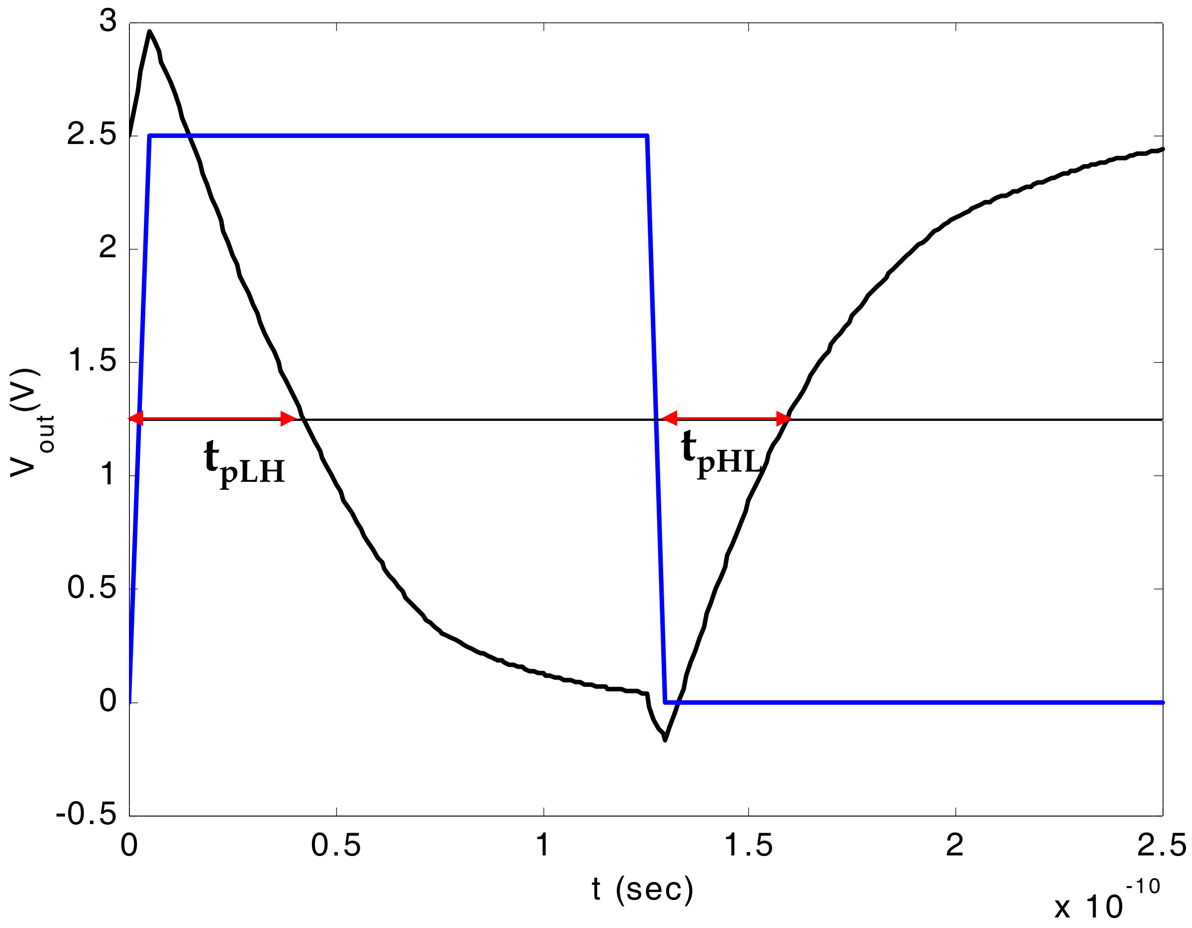
- 增加器件的宽长比会同时增加导电因子和器件的栅电容和漏区电容
- 对于固定的大负载电容可以通过增加器件尺寸提高速度
- 对于小负载，反相器速度不会随着尺寸出现明显增加

$$t_{\mathrm{p}} = \frac{t_{\mathrm{pHL}} + t_{\mathrm{pLH}}}{2}$$

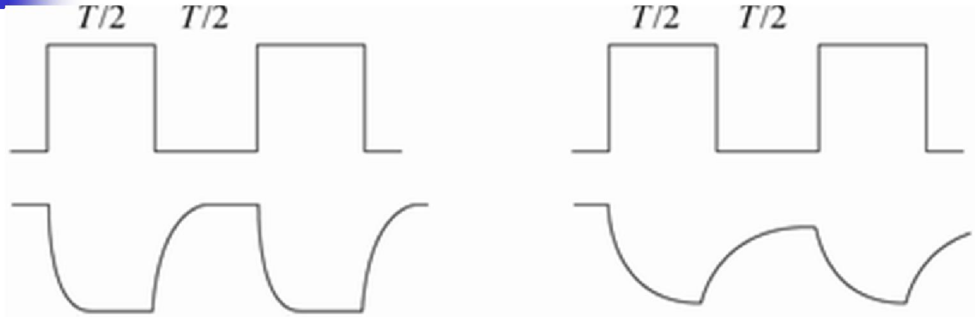
$$t_{\mathrm{pHL}} = \frac{C_{\mathrm{L}} \Delta V_{\mathrm{HL}}}{I_{\mathrm{av,HL}}} \approx \tau_{\mathrm{f}} \frac{1}{\left(1-\alpha_{\mathrm{N}}\right)^2}$$

$$t_{\mathrm{pLH}} = \frac{C_{\mathrm{L}} \Delta V_{\mathrm{LH}}}{I_{\mathrm{av,LH}}} \approx \tau_{\mathrm{r}} \frac{1}{\left(1-\alpha_{\mathrm{P}}\right)^2}$$

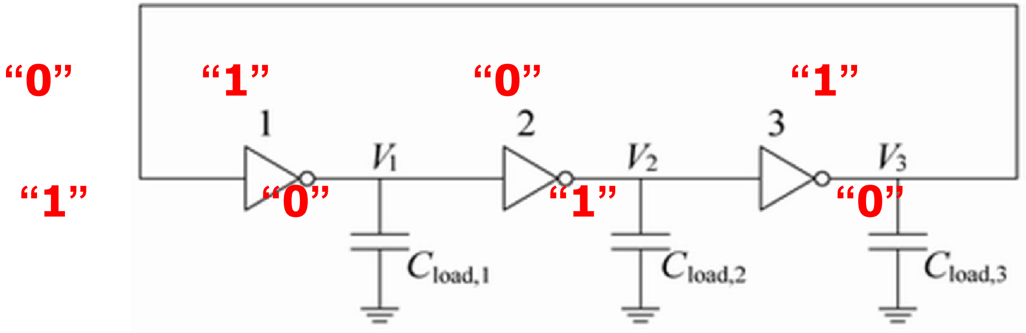
瞬态响应：仿真波形



电路的最高工作频率



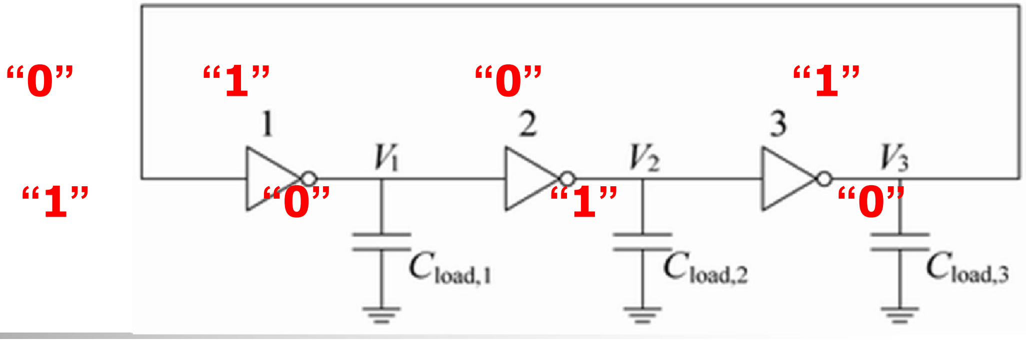
$$f_m = \frac{1}{2\max(t_r, t_f)}$$

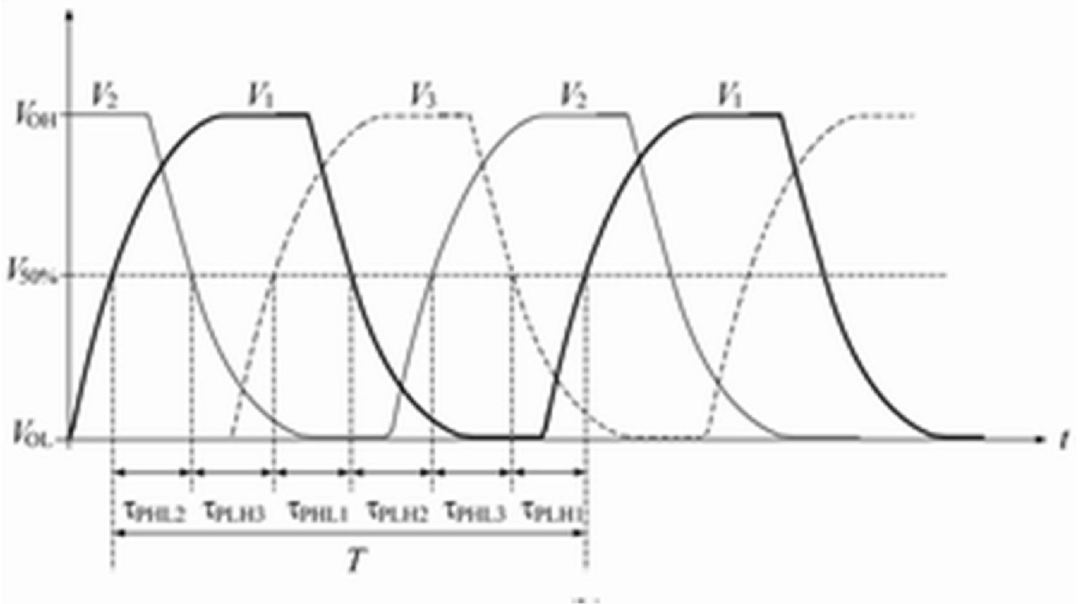


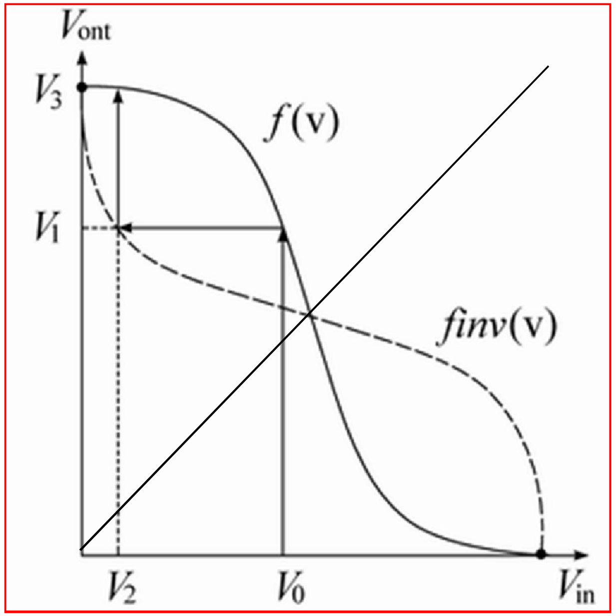
用环形振荡器测量
传输延迟时间

$$t_p = \frac{1}{2nf}$$

环形振荡器的频率







$$T = t_{\mathrm{PHL2}} + t_{\mathrm{PLH3}} + t_{\mathrm{PHL1}} + t_{\mathrm{PLH2}} + t_{\mathrm{PHL3}} + t_{\mathrm{PLH1}}$$

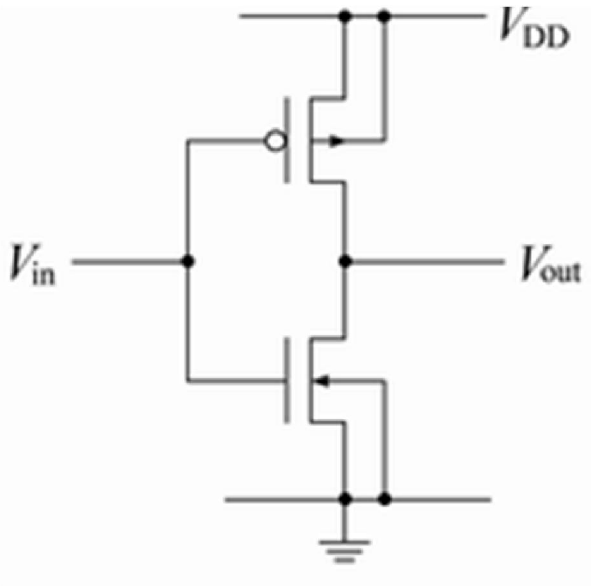
$$= 2\left(t_{\mathrm{p1}} + t_{\mathrm{p2}} + t_{\mathrm{p3}}\right) = 6t_{\mathrm{p}}$$

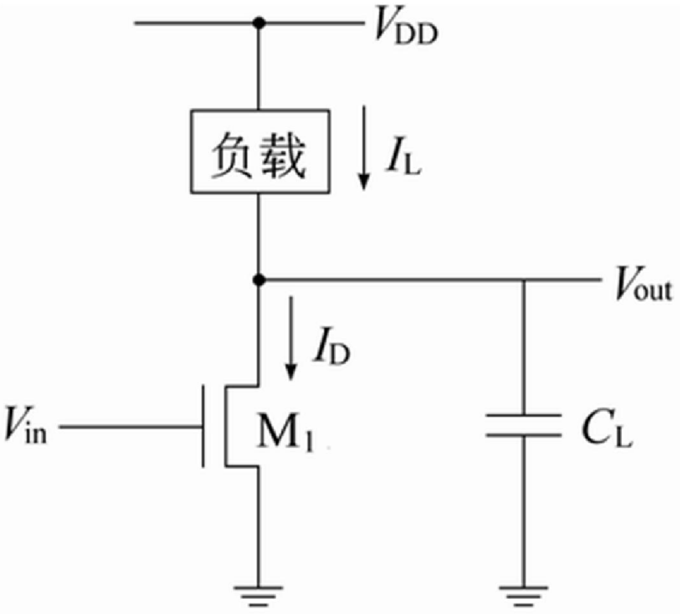
$$f = \frac{1}{6t_{\mathrm{p}}}$$

CMOS反相器

- **4.1 CMOS反相器的直流特性**
- **4.2 CMOS反相器的瞬态特性**
- **NMOS反相器**

CMOS和NMOS反相器结构比较

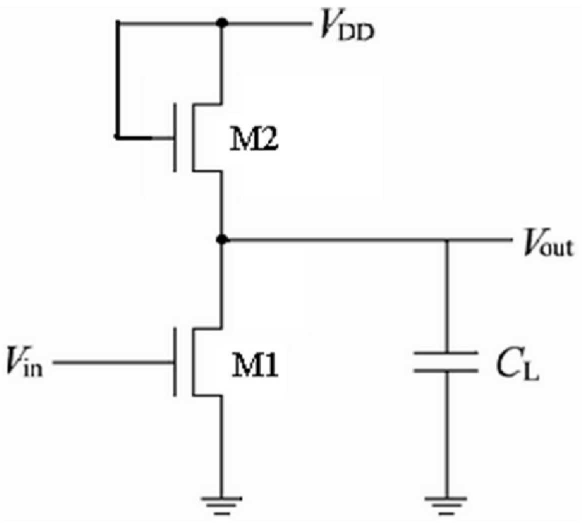




2个可控开关

开关 **+ 常导通负载**

1、饱和负载NMOS反相器



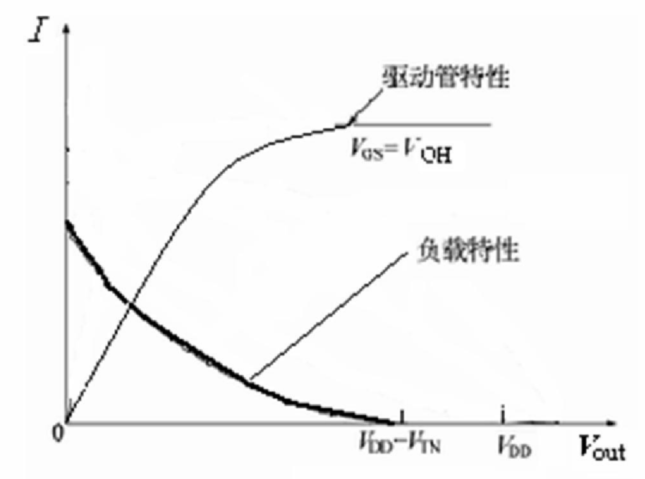
2个增强型NMOS组成,
M1是可控开关,驱动管,
负载管**M2**总是在饱和区

负载特性

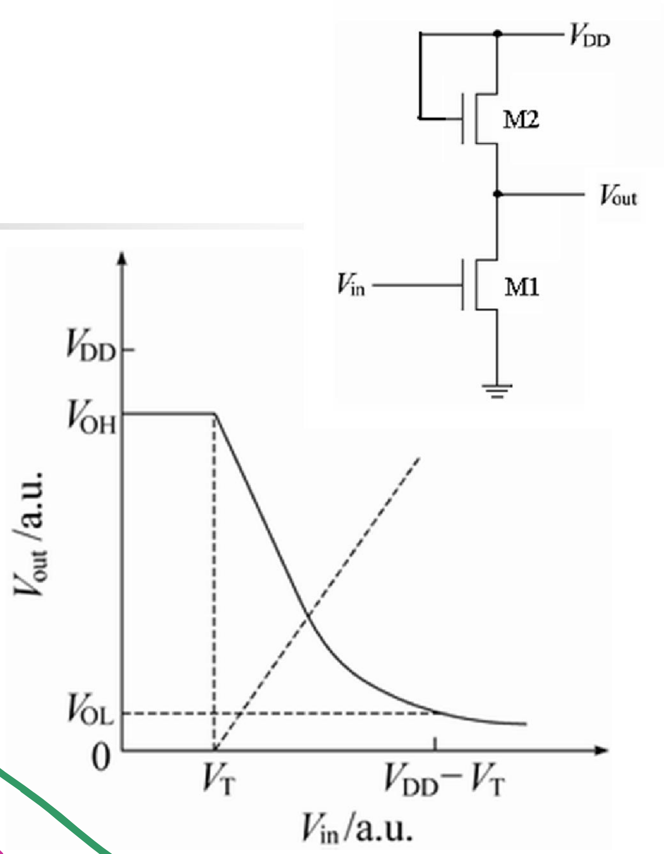
$$V_{G2}=V_{D2}=V_{DD}$$

$$I_{D2}=K_2\left(V_{DD}-V_T-V_{out}\right)^2$$

$$\text{当}V_{out}=V_{DD}-V_T，I_{D2}=0$$



饱和负载反相器的直流电压传输特性



1. $0 \le V_{in} \le V_T$ **M1截止，M2饱和**

$I_{D1}=I_{D2}=0$, $Vout=V_{OH}=V_{DD}-V_T$ 阈值损失

2. $0 < V_{in} - V_T \le V_{out}$ **M1饱和，M2饱和**

$$K_1(V_{in}-V_T)^2 = K_2(V_{DD}-V_T-V_{out})^2$$

$$V_{out} = V_{OH} - \sqrt{K_r}(V_{in}-V_T)$$

3. $V_{IH} \ge V_{in} > V_{out} + V_T$ **M1线性，M2饱和**

$$K_1\left[(V_{in}-V_T)^2-(V_{in}-V_T-V_{out})^2\right]=K_2(V_{DD}-V_T-V_{out})^2$$

$$V_{OL} \approx \frac{V_{OH}^2}{2K_r(V_{OH}-V_T)}$$

要求$Kr >> 1$，有比电路

输出低电平时有直流电流 $I_{on} = K_2(V_{DD}-V_T-V_{OL})^2$

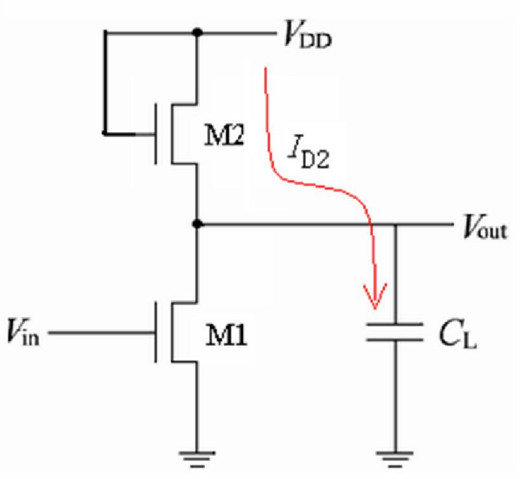
饱和负载反相器的上升时间

上升过程

$$C_L \frac{dV_{out}}{dt} = K_2 (V_{DD} - V_T - V_{out})^2$$

归一化

$$u = \frac{V_{out}}{V_{OH}} = \frac{V_{out}}{V_{DD} - V_T}$$

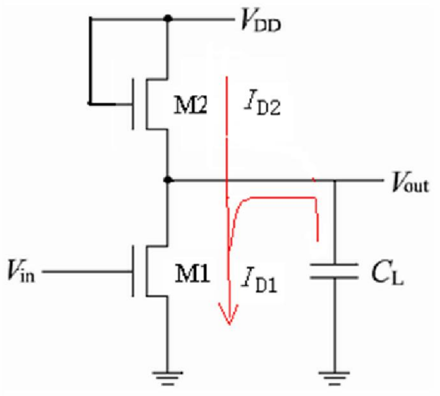


$$t_r = \tau_r \left(\frac{1}{1-u_2} - \frac{1}{1-u_1} \right)$$

$$\tau_r = \frac{C_L}{K_2 V_{OH}} = \frac{C_L}{K_2 (V_{DD} - V_T)}$$

$$u_1 = 0.1, \quad u_2 = 0.9, \quad t_r \approx 8.9\tau_r$$

饱和负载反相器的下降时间



下降过程 $-C_L \dfrac{dV_{out}}{dt} = I_{D1} - I_{D2}$

忽略**M2**的电流 $-C_L \dfrac{dV_{out}}{dt} = I_{D1}$

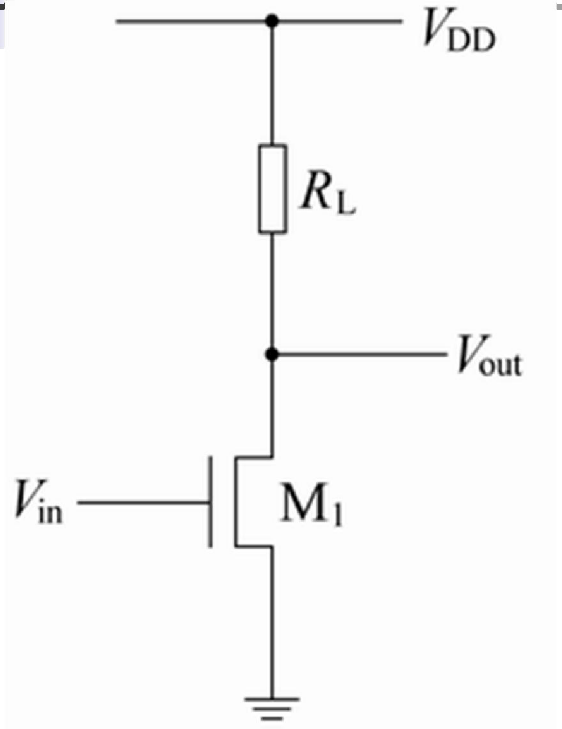
归一化 $u = \dfrac{V_{out}}{V_{OH} - V_T}$

$u \geq 1$，M1饱和 $-\dfrac{du}{dt} = \dfrac{1}{\tau_f}$, $\tau_f = \dfrac{C_L}{K_1\left(V_{OH} - V_T\right)}$

$u < 1$，M1线性 $-\tau_f \dfrac{du}{dt} = 2u - u^2$

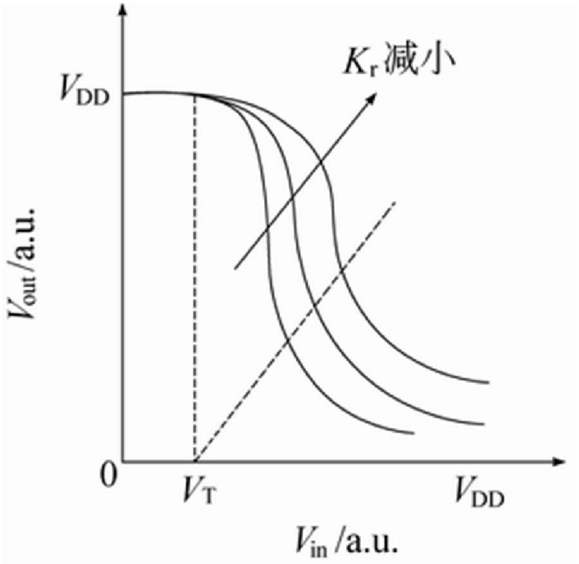
$$t_f = \tau_f\left[\left(u_1 - 1\right) + \frac{1}{2}\ln\left(\frac{2 - u_2}{u_2}\right)\right] \qquad u_1 = \frac{0.9V_{OH}}{V_{OH} - V_T} \qquad u_2 = \frac{0.1V_{OH}}{V_{OH} - V_T}$$

2、电阻负载NMOS反相器



一个多晶硅电阻做负载

负载特性



电阻负载反相器的VTC

1. $0 \le V_{in} \le V_{TN}$ **M1截止**

$I_{D1}=I_R=0$, $Vout=V_{OH}=V_{DD}$

2. $0 < V_{in} - V_T \le V_{out}$ **M1饱和**

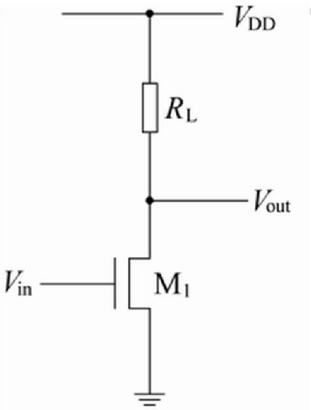
$$\frac{V_{DD}-V_{out}}{R_L} = K_1 \left(V_{in}-V_T\right)^2$$

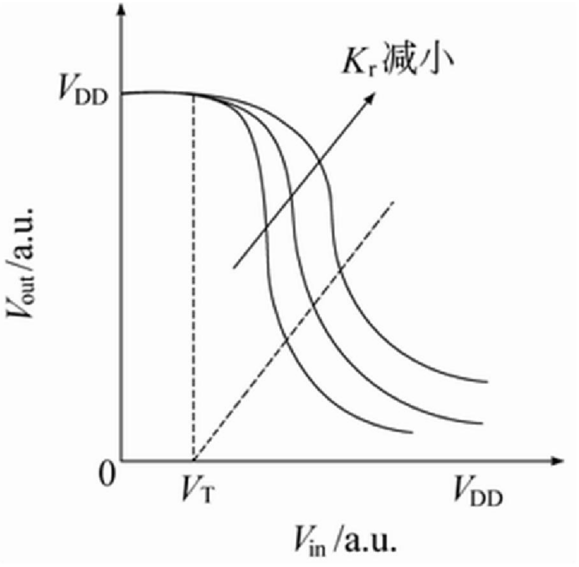
$$V_{out} = V_{DD} - \frac{K_r}{V_{DD}}\left(V_{in}-V_T\right)^2, \quad K_r = V_{DD} R_L K_1$$

3. $V_{DD} \ge V_{in} > V_{out} + V_T$ **M1线性**

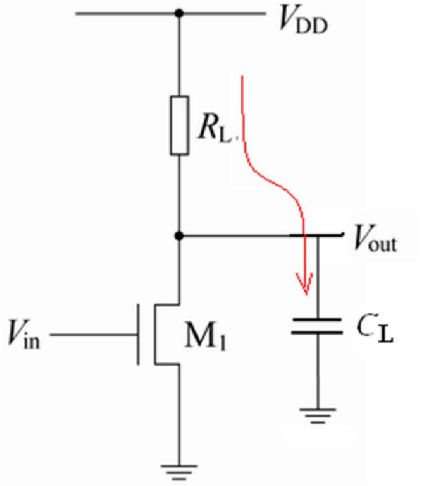
$$\frac{V_{DD}-V_{out}}{R_L} = K_1\left[\left(V_{in}-V_T\right)^2-\left(V_{in}-V_T-V_{out}\right)^2\right]$$

$$V_{OL} \approx \frac{V_{DD}^2}{2K_r\left(V_{DD}-V_T\right)} \qquad I_{on} = \frac{V_{DD}-V_{OL}}{R_L} \approx \frac{V_{DD}}{R_L}$$





电阻负载反相器的瞬态特性



上升过程

$$C_L \frac{dV_{out}}{dt} = \frac{V_{DD} - V_{out}}{R_L}$$

$$t_r = \tau_r \left[\ln(1-u_1) - \ln(1-u_2) \right] = 2.2\tau_r$$

$$u_1 = 0.1, \quad u_2 = 0.9, \quad \tau_r = R_L C_L$$

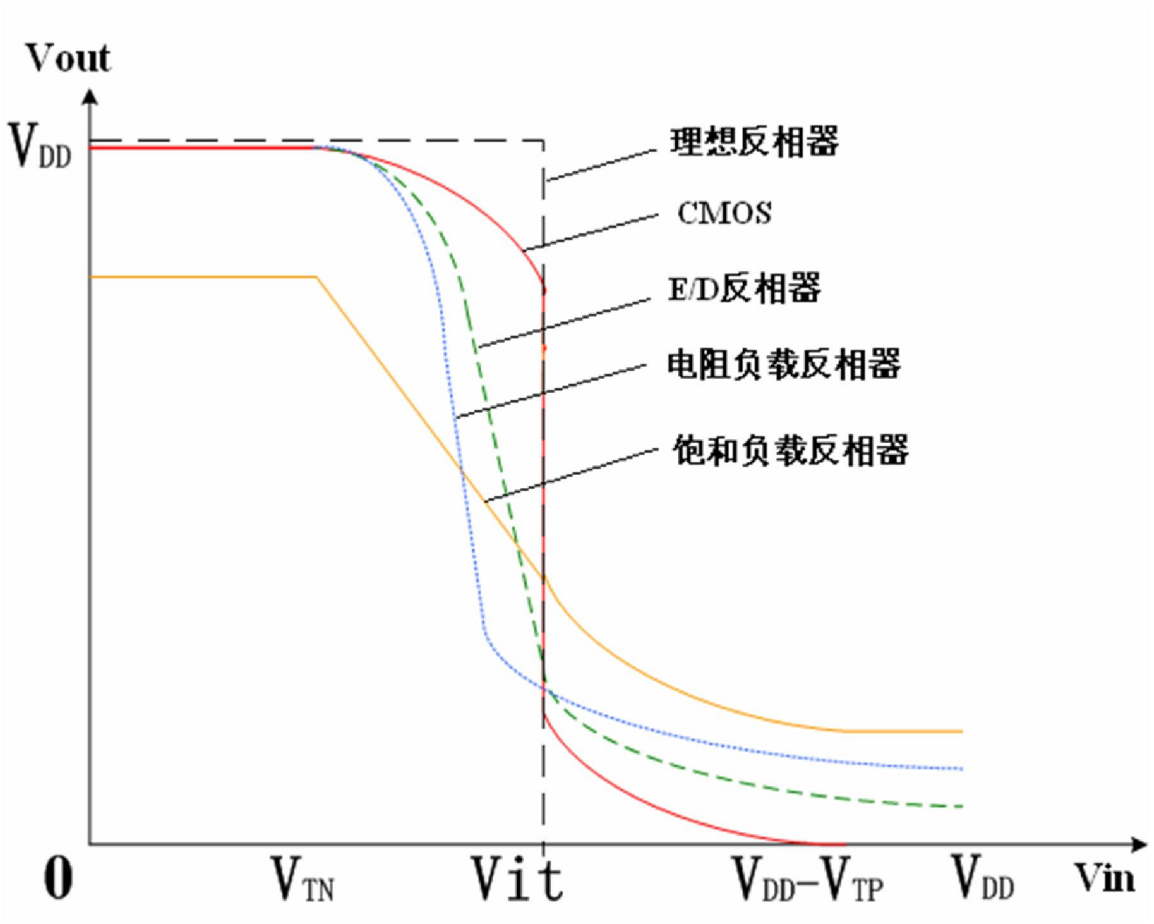
下降过程，忽略负载电流与**CMOS**反相器相同

$$t_f = \tau_f \left[\frac{\alpha_N - 0.1}{(1-\alpha_N)^2} + \frac{1}{2(1-\alpha_N)} \ln\left(\frac{1.9 - 2\alpha_N}{0.1} \right) \right]$$

CMOS和NMOS反相器直流特性比较

	饱和负载	耗尽型负载	电阻负载	**CMOS**
V_{OH}	V_{DD}-V_T	V_{DD}	V_{DD}	V_{DD}
V_{OL}	$\frac{V_{OH}^2}{2K_r(V_{OH}-V_T)}$	$\frac{V_{TD}^2}{2K_r(V_{DD}-V_T)}$	$\frac{V_{DD}^2}{2K_r(V_{DD}-V_T)}$	0
K_r	K_1/K_2	K_E/K_D	$V_{DD}R_LK_1$	K_N/K_P
K_r=5	V_{OL}= 0.53	0.1	0.63	0
I_{on}	$K_2(V_{DD}-V_T-V_{OL})^2$	$K_DV_{TD}^2$	V_{DD}/R_L	0

直流电压传输特性比较


Vout
VDD
理想反相器
CMOS
E/D反相器
电阻负载反相器
饱和负载反相器
0
VTN
Vit
VDD-VTP
VDD
Vin

CMOS和NMOS反相器瞬态特性比较

- 上升,下降时间比较

参数	饱和负载	耗尽负载	电阻负载	CMOS
t_r	$8.9\tau_r$	$0.9\tau_r$	$2.2\tau_r$	$1.8\tau_r$
t_f	$1.5\tau_f$	$1.8\tau_f$	$1.8\tau_f$	$1.8\tau_f$
t_r / t_f	$4.5Kr$	$3.1\ Kr$	$1.2\ Kr$	1

$V_{TN} = -V_{TP} = V_{TE} = 0.2V_{DD}$， $V_{TD} = 0.4V_{DD}$

注意：不同电路的τ_r和τ_f是不同的

CMOS电路的优点

- 无比电路，具有最大的逻辑摆幅
- 在低电平状态不存在直流导通电流
- 静态功耗低
- 直流噪声容限大
- 采用对称设计获得最佳性能