

§ 13.2 早期宇宙中的粒子与核相互作用

1. 早期宇宙中的粒子相互作用

$t=10^{-12}\text{s}$ 时, $T\sim 10^{16}\text{K}$, $E\sim 1000\text{GeV}$ 。

除了可能是禁闭的夸克之外, 所有已知的粒子都可以在这个能量下产生出来, 并且都与辐射场达到热平衡。各种粒子的产生率和粒子数密度都只是正比于粒子的内秉自由度。

$$\gamma + \gamma \leftrightarrow e^+ + e^-; \quad \gamma + \gamma \leftrightarrow \mu^+ + \mu^-; \quad \gamma + \gamma \leftrightarrow \tau^+ + \tau^-$$

按照黑体辐射的描写, 光子体系的能量和数量密度为:

$$u(E)dE = \frac{8\pi E^3}{(hc)^3} \frac{1}{e^{E/kT} - 1} dE \quad (13.2-1)$$

$$n(E)dE = \frac{u(E)}{E} dE = \frac{8\pi E^2}{(hc)^3} \frac{1}{e^{E/kT} - 1} dE \quad (13.2-2)$$

对能量积分

$$\rho_\gamma = 4.7 \times 10^3 T^4 \quad \text{eV/m}^3 \quad (13.2-3)$$

$$N_\gamma = 2.0 \times 10^7 T^3 \quad \text{photons/m}^3 \quad (13.2-4)$$

对于当今**2.7 K**的背景辐射，光子的平均能量为**0.001 eV**，数密度为 **$4 \times 10^8/\text{m}^3=400/\text{cm}^3$** ，而能量密度为约 **$0.25 \text{ eV}/\text{cm}^3$** 。

现今宇宙中的物质密度：

明物质 $\rho_0 \approx 3 \times 10^{-31} \text{ g}/\text{cm}^3$

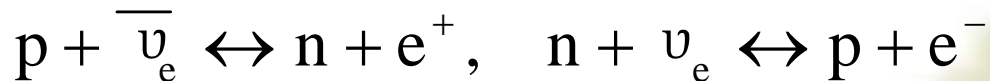
加上暗物质 $\rho_0 \approx 6 \times 10^{-31} \text{ g}/\text{cm}^3 \approx 0.4 \text{ 核子}/\text{m}^3$

核子数密度比背景光子小 **10^9** 倍

人们相信现在的宇宙只是由物质构成的，而没有自然存在的反物质，这显然与初始宇宙的状况不一样。目前估计这种不平衡是由**CP**不守恒造成的。在 $t < 10^{-6}\text{s}$ 时，正反核子一方面湮没成光子，另一方面又从其他途径产生正反核子（或正反夸克）。产生的过程由于**CP**不守恒造成正物质多于反物质。

$$t > 10^{-6}\text{s}, \quad T \leq 10^{13}\text{ K } (E \leq 1\text{ GeV})$$

正反核子湮没，且不再能与它的逆过程平衡。湮没之后剩余少量的正物质。由于存在一定丰度的轻子，弱作用可以发生，使得质子和中子可以相互转化而达成平衡



$$t=10^{-2}\text{s}, T=10^{11}\text{K} (E=10\text{ MeV})$$

带电轻子中只有正反电子存在，正反 μ 子和 τ 子都湮没了，且不再由辐射光子产生。然而，通过弱中性作用，可以产生所有的中微子：

$$e^+ + e^- \leftrightarrow Z^0 \leftrightarrow \nu_e + \bar{\nu}_e, \quad e^+ + e^- \leftrightarrow Z^0 \leftrightarrow \nu_\mu + \bar{\nu}_\mu, \quad e^+ + e^- \leftrightarrow Z^0 \leftrightarrow \nu_\tau + \bar{\nu}_\tau,$$

$$u(E)dE = g_i \frac{4\pi E^3}{(hc)^3} \frac{1}{e^{E/kT} + 1} dE \quad (13.2-5)$$

$$g_e = 4 \quad (\text{2个自旋态, 正反粒子态})$$

$$g_{\nu_i} = 2 \quad (\text{正反粒子态, } i = e, \mu, \tau)$$

$$N_e = \frac{3}{2} N_\gamma, \quad \rho_e = \frac{7}{4} \rho_\gamma, \quad N_\nu = \frac{3}{4} n_\nu N_\gamma, \quad \rho_\nu = \frac{7}{8} n_\nu \rho_\gamma \quad (13.2-6)$$

n_ν 是中微子的种类数（考虑 e, μ, τ 时， $n_\nu=3$ ）

在温度 T 下的平衡混合物中，按照玻耳兹曼分布可得：

$$\frac{N_n}{N_p} = e^{-(m_n - m_p)/kT} \quad (13.2-7)$$

在 $kT=10 \text{ MeV}$ 时 ($t \approx 0.01\text{s}$)，可以算得 $N_n / N_p = 0.88$ 。

$t=1\text{s}$ 时 ($T=10^{10} \text{ K}$, $E=1 \text{ MeV}$)

中微子反应不再能发生，宇宙进入了中微子退耦合时代。中微子随着宇宙自由膨胀，不受核反应的影响。随后，由两个光子产生正负电子的过程也终止了，而正负电子湮没成光子引起光子数密度相对增加，并导致光子体系的温度高于中微子体系的温度，其比例关系为：

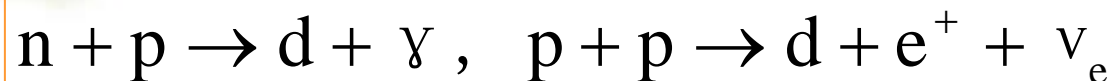
$$\frac{T_\gamma}{T_\nu} = \left(\frac{11}{4}\right)^{1/3} \approx 1.4 \quad (13.2-8)$$

计算中假定有三代轻子。由此可知现在宇宙中背景中微子的温度大约是**2 K**。而数密度也与背景光子同数量级。

在正负电子湮没之后，宇宙中剩下大量的光子和中微子，相当于光子的 10^{-9} 比例的电子和质子，比质子略少的中子（有的理论预言也有极少量的较重的原子核）。这样就终止了粒子反应的阶段而开始了核反应的阶段。

2. 原初核合成

在质子中子形成后的第一批核反应应当是



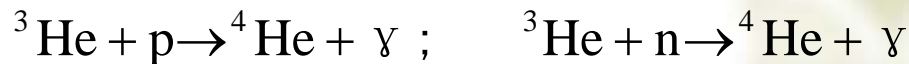
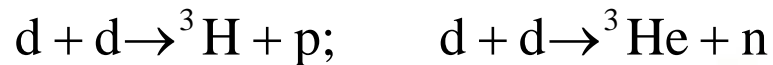
在温度很高时，由于光子数密度非常大，氘核产生后立即分解，无法积累。分解氘核所需的光子能量是氘的结合能 $E_0=2.225\text{MeV}$ 。但是，我们不能以光子的平均能量来考虑。按照13.2-1式，光子的能量分布有一个指数衰减的尾巴，在平均能量很低时，也有高能量的成分。由于光子数密度比核子数密度大 10^9 倍，因此即便是很小比例的高能成分也足以造成氘的及时分解。为了确定氘开始积累的时间，我们积分计算在一定温度 T 下能量高于 E_0 的光子的数目与总数之比，结果为：

$$f(E > E_0) = 0.42e^{-E_0/kT} \left[\left(\frac{E_0}{kT} \right)^2 + 2 \left(\frac{E_0}{kT} \right) + 2 \right] \quad (13.2-9)$$

其中假定了。由于中子数密度低于质子数密度，由n+p反应生成氘的概率由中子数目决定。按照式13.2-7，中子数与质子数之比与温度有关。由前面的讨论知道，在一定的温度之下，n-p之间的转换会终止。这个使中质比“冻结”下来的温度是 $9 \times 10^9 \text{K}$ ，对应的 $N_n / N_p \approx 0.2$ ，而时间是约3s 。

显然，如果如前所述，核子与光子之比是 10^{-9} 的话，为了使产生的氘及时分解成核子，需要的能量高于E0的光子数目与总光子数目之比应为。由13.2-9式可知，此时对应的温度是 $T = 9 \times 10^8 \text{K}$ ，而时间是 $t \approx 250\text{s}$ 。在这个时间之后，光子场不足以使所有的氘核分解，从而开始积累氘核素。

一旦形成了足够量的氘，其它的核反应就成为可能。

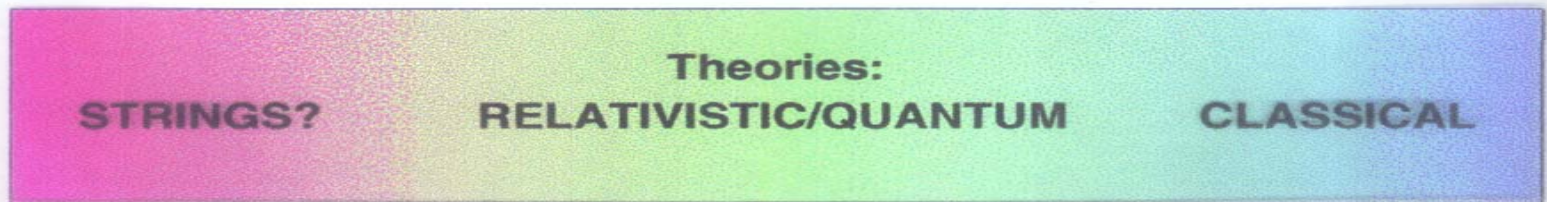
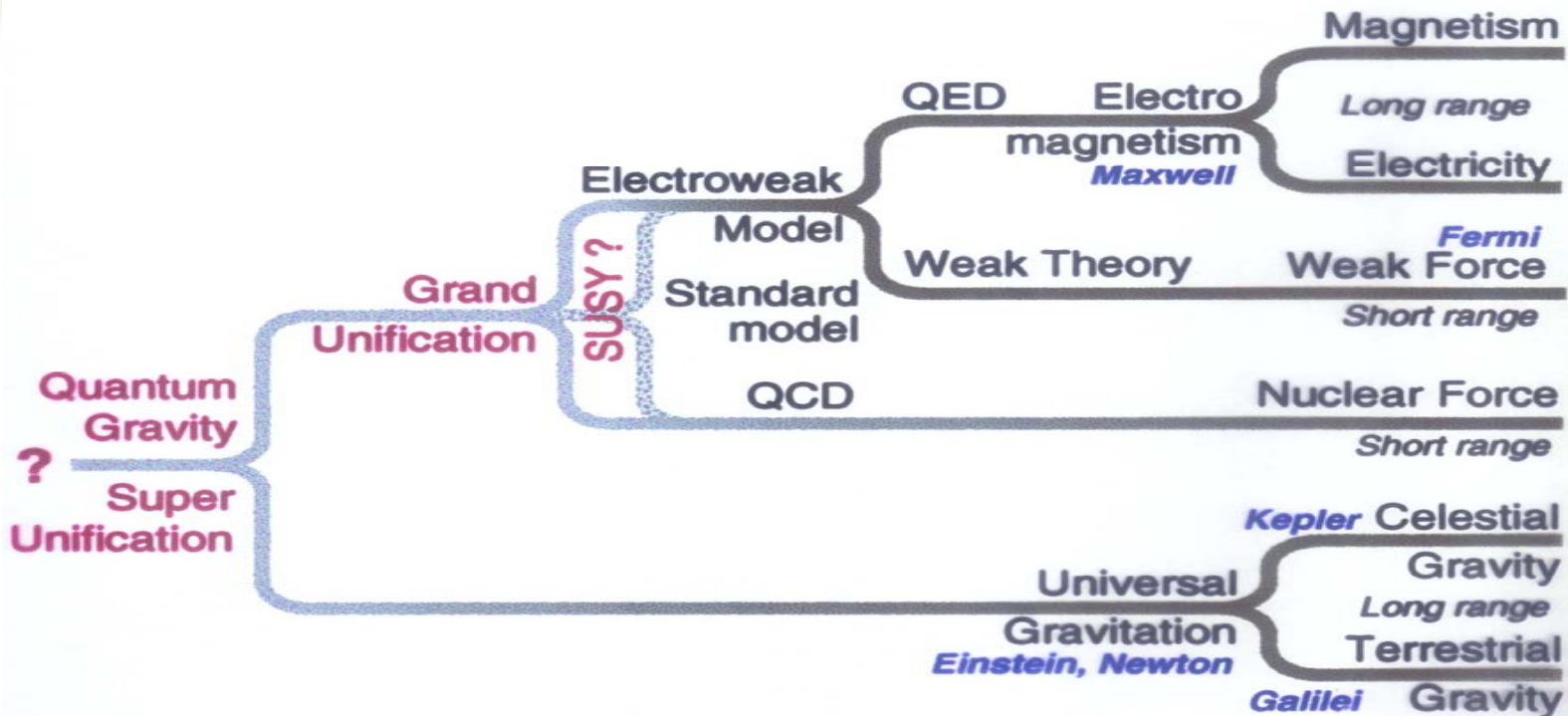
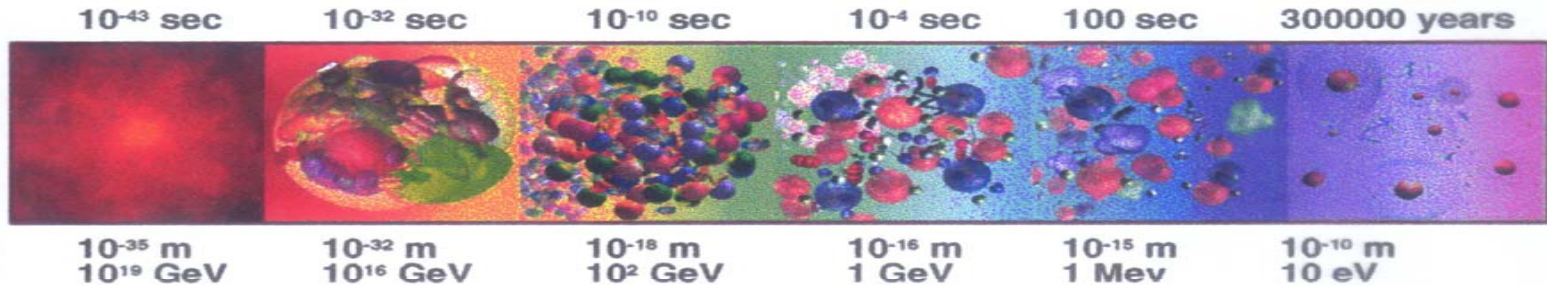


主要是生成 ${}^4\text{He}$

这样，在**250s**之后，所有的中子都将变成**He**原子核。考虑到从**3s**到**250s**之间中子的自发 β 衰变，可以推出相对丰度 $N_{\text{He}} / N_{\text{p}} = 0.061$ 。这样， **^4He** 的相对质量丰度为约**0.24**。如果不考虑后来在星体中的核反应，则 **^4He** 的丰度应当从**250s**一直保持到现在。实际上，现在从各种天体物质中观察到的结果与上述的数字非常接近。然而，前面的推导实际上与我们取的原初核子和光子之比（取决于**CP**破缺的程度）有密切关系，也依赖于中子 β 衰变的半衰期和模型中中微子有多少代。

在稳定的轻原子核（如 **^2H** 、 **^4He** 、 **^7Li** ）形成后，宇宙继续膨胀和冷却直到光子退耦合，也就是光子能量低于原子的电离能。这时，稳定的原子就形成了。原子形成期大约是**700000**年的时候（温度约**3000K**）。此后，光子体系也就象中微子体系一样，不再参与反应，随宇宙自由膨胀和冷却，直到现在的**2.7 K**。

Summary



Quantum gravity era

10^{-43} s

Gravity separates as a force, the other forces remain as one (Grand Unification)



$t < 10^{-43}$ s : The Big Bang

The universe is considered to have expanded from a single point with an infinitely high energy density (infinite temperature). Is there a meaning to the question what existed before the big bang?

$t \approx 10^{-43}$ s, 10^{32} K (10^{19} GeV, 10^{-34} m) : Gravity "freezes" out

All particle types (quarks, leptons, gauge bosons, and undiscovered particles e.g. Higgs, sparticles, gravitons) and their anti-particles are in a thermal equilibrium (being created and annihilated at equal rate). These coexist with photons (radiation). Through a phase transition gravity "froze" out and became distinct in its action from the weak, electromagnetic and strong forces. The other three forces could not be distinguished from one another in their action on quarks and leptons. This is the first

Grand unification era

10^{-35} s

Inflation ceases, expansion continues
Grand Unification breaks. Strong and electroweak
forces become distinguishable



$t \approx 10^{-36}$ s, 10^{27} K (10^{16} GeV, 10^{-32} m) : Inflation

The rate of expansion increases exponentially for a short period. The universe doubled in size every 10^{-36} s. Inflation stopped at around 10^{-32} s. The universe increased in size by a factor of 10^{26} . This is equivalent to an object the size of a proton swelling to 10^{26} light years across. The whole universe is estimated to have had a size of $\sim 10^{-26}$ m at the end of the period of inflation. However the presently visible universe was only 3 m in size after inflation. This solves the problems of 'horizon' (how is it possible for two opposing parts of the present universe to be at the same temperature when they cannot have interacted with each other before recombination) and 'flatness' (density of matter is close to the critical density).

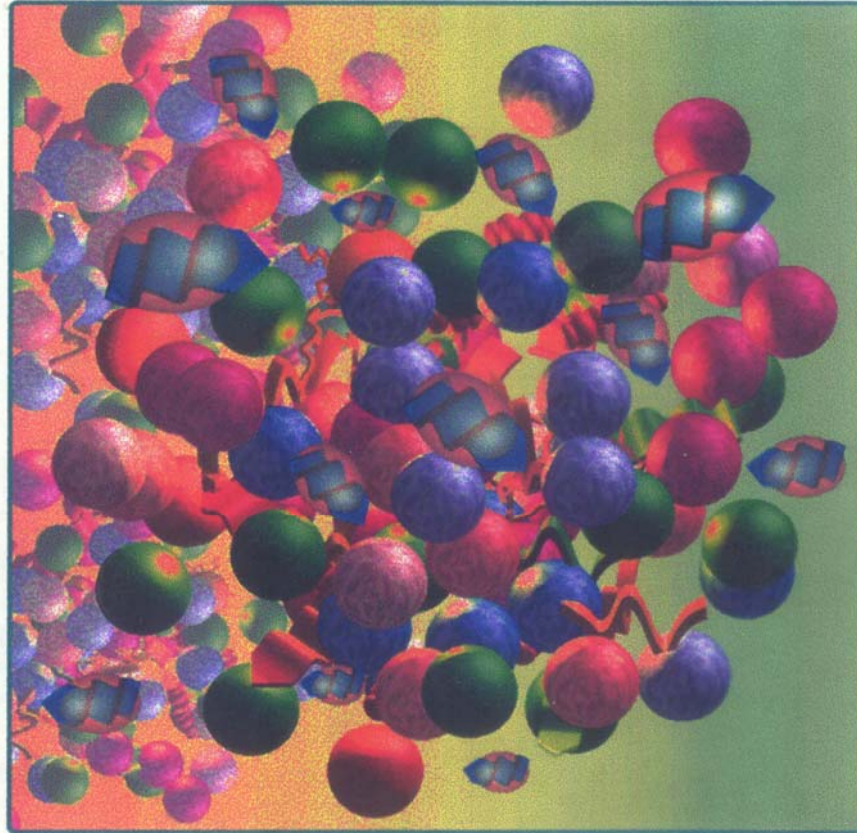
$t \approx 10^{-32}$ s : Strong forces freezes out

Through another phase transition the strong force "freezes" out and a slight excess of matter over anti-matter develops. This excess, at a level of 1 part in a billion, is sufficient to give the presently observed predominance of matter over anti-matter. The

Electroweak era

10^{-10} s

Electroweak force splits

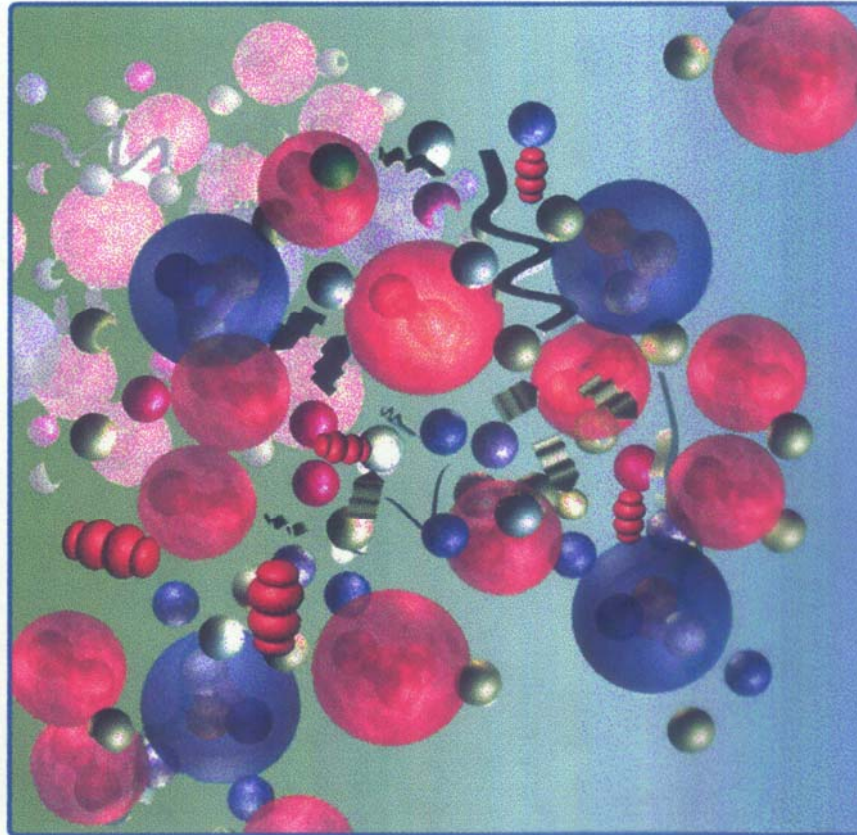


$t \sim 10^{-10}$ s, 10^{16} K (100 GeV, 10^{-16} m) : Electromagnetic and Weak Forces separate

The energy density corresponds to that at LEP. As the temperature fell the weak force "freezes" out and all four forces become distinct in their actions. The antiquarks annihilate with the quarks leaving a residual excess of matter. W and Z bosons decay. In

Protons and neutrons form 10^{-4} s

Quarks combine to make protons and neutrons



$t = 10^{-4}$ s, 10^{13} K (1 GeV, 10^{-16} m) : Protons and Neutrons form

The universe has grown to the size of our solar system. As the temperature drops quark-antiquark annihilation stops and the remaining quarks combine to make protons and neutrons.

$t = 1$ s, 10^{10} K (1 MeV, 10^{-13} m) : Neutrinos decouple

Nuclei are formed 100 s

Protons and neutrons combine to form helium nuclei



t = 3 minutes, 10^9 K (0.1 MeV, 10^{-12} m): Nuclei are formed

The temperature is low enough to allow nuclei to be formed. Conditions are similar to those that exist in stars today or in thermonuclear bombs. Heavier nuclei such as deuterium, helium and lithium soak up the neutrons that are present. Any remaining neutrons decay with a time constant of ~ 1000 seconds. The neutron-proton ratio is now 13:87. The bulk constitution of the universe is now in place consisting essentially of protons (75%) and helium nuclei. The temperature is still too high to

Atoms and light era 300000 years

The Universe becomes transparent and fills with light



$t = 300\,000$ years, 6000 K (0.5 eV, 10^{-10} m) : Atoms are created

Electrons begin to stick to nuclei. Atoms of hydrogen, helium and lithium are created. Radiation is no longer energetic

Galaxy formation 1000 million years

Galaxies begin to form



$t = 10^8$ years, 18 K : Galaxy Formation

Local mass density fluctuations act as seeds for stellar and galaxy formation. The exact mechanism is still not understood. Nucleosynthesis, synthesis of heavier nuclei such as carbon up to iron, starts occurring in the thermonuclear reactors that are stars. Even heavier elements are synthesized and dispersed in the brief moment during which stellar collapse and