What is Left of the Multiplier Accelerator?

Olivier J. Blanchard

The American Economic Review, Vol. 71, No. 2, Papers and Proceedings of the Ninety-Third Annual Meeting of the American Economic Association. (May, 1981), pp. 150-154.

Stable URL:
http://links.jstor.org/sici?sici=0002-8282%28198105%2971%3A2%3C150%3AWILOTM%3E2.0.CO%3B2-A

The American Economic Review is currently published by American Economic Association.

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at http://www.jstor.org/about/terms.html. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at http://www.jstor.org/journals/aea.html.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is an independent not-for-profit organization dedicated to creating and preserving a digital archive of scholarly journals. For more information regarding JSTOR, please contact support@jstor.org.

http://www.jstor.org/
Wed Dec 6 08:01:49 2006

What is Left of the Multiplier Accelerator?

By OLIVIER J. BLANCHARD*

One of the few undisputed facts in macroeconomics is that output is hump shaped, or more precisely that the distribution of weights of the moving average representation of the deviation of quarterly output from an exponential trend has a hump shape. The first eight weights of the distribution are given in Table 1, column 1. Nearly equivalently, output is well characterized by the following $AR(2)$:

$$Y_t = \underset{(16.4)}{1.34}\ Y_{t-1} - \underset{(-5.21)}{.42}\ Y_{t-2} + \varepsilon_t$$

Y: logarithm of real quarterly *GNP* minus linear trend; sample period: 47–3 to 78–4.

This implies that, following a movement of output from its equilibrium value this period, we expect a movement of output further away from equilibrium for three more quarters before it returns to equilibrium. It also implies that, given only the past history of output, we can predict—some—turning points from expansion to recession (i.e., sequences $EY_{t+i} > EY_{t+i-1}$; $EY_{t+i} > EY_{t+i+1}$) or the reverse; this would not be the case if the best representation of output was a first-order autoregressive process for example.

I. The Multiplier Accelerator

The traditional explanation of the hump shape relies on the dynamics of *private spending* and the combination of the multiplier and accelerator mechanisms.

In its original form (see Paul Samuelson), it is given by

$$C_t = \alpha Y_{t-1}; \quad I_t = \gamma(Y_{t-1} - Y_{t-2})$$

$$Y_t = C_t + I_t + G_t \Rightarrow$$

$$Y_t = (\alpha+\gamma) Y_{t-1} - \gamma Y_{t-2} + G_t$$

where G is autonomous expenditures. This has the required implication for $(\alpha+\gamma) > 1$. In this case white noise disturbances in G generate a hump shape for output.

The large macro-econometric models also generate a hump-shaped response of output, although not to white noise but to serially correlated disturbances in G. This is shown in Table 1, columns (2) to (4), for the *MPS* model. Although interest rates and prices are endogenous, the hump shaped response of output comes from the *IS* dynamics: interest rates and price movements only dampen the effect of G. These *IS* dynamics in turn are explicitly constructed around the multiplier accelerator mechanism (see Carol Corrado for the *MPS* model).

If we consider column (3), it is characterized by substantial anticipated movements in private spending. Although the initial movement in period 1 is in response to the unanticipated shock in G and therefore unanticipated itself, the movements in period 2 and following are anticipated. It is the existence of such anticipated movements in either consumption or investment which has recently come under attack. It has been argued that, *given interest rates* and tax rates, most movements in consumption and investment are due to new information and that there cannot be large anticipated movements in consumption or investment. If the argument is correct, the multiplier accelerator and, with it, the *IS* dynamics of large macro-econometric models are misleading and we should look elsewhere for an explanation of the hump shape.

The rest of the paper reviews the theoretical arguments. The next sections present first the case for the prosecution and then for the defense.

*Harvard University. I thank Andy Abel, Ben Friedman, Francesco Giavazzi, Robert Shiller, and Larry Summers for discussions, the National Science Foundation and the Sloan Foundation for financial assistance.

TABLE 1—RESPONSE OF REAL *GNP*

Quarters	*Y* (1)	*G* (2)	*A* (3)	*Y* (4)
1	1.00	1.00	.16	1.16
2	1.32	.90	.43	1.33
3	1.47	.81	.64	1.45
4	1.50	.73	.82	1.55
5	1.30	.65	.76	1.41
6	.99	.59	.72	1.31
7	.80	.53	.64	1.17
8	.79	.47	.36	.83

Note: Col. (1): Estimated response of *GNP* to a one-time deviation of 1 of its own disturbance in quarter 1. (*MA* representation derived from an *AR*(12) estimated on 1947–3 to 1978–4.) Cols. (2), (3), (4): Simulated response of *GNP*(*Y*) and private spending (*A*) to a one-time deviation of 1 in ε, with $G=.9G_{-1}+\varepsilon$(G: government expenditures) in the *MPS* model.

II. An Epidemic of Martingales

For *consumption*, the point was made by Robert Hall. Assuming that an individual maximizes expected lifetime utility, he will always act so as to equalize current marginal utility and discounted expected future marginal utility. Formally, if C is consumption, δ the discount rate, r the interest rate assumed known and constant for our purposes, and Ω_t his information set at time t:

$$(1)\qquad U'(C_t)=\frac{1+r}{1+\delta}E[U'(C_{t+1})|\Omega_t]$$

Under the further assumption that utility is quadratic (or else as an approximation), this gives

$$E\left[\frac{1+r}{1+\delta}C_{t+1}-C_t|\Omega_t\right]=0$$

The implication is that even if δ is different from r, anticipated movements in income, as they belong to Ω_t, do not lead to anticipated movements in consumption. Furthermore, if $\delta=r$, there are no anticipated movements in consumption: consumption follows a martingale. Otherwise, if δ is different from r, it follows a sub- or a supermartingale.

As a sum of (sub, super) martingales is a (sub, super) martingale, aggregate consumption, excluding those who enter and leave the consumption pool each period, is also a (sub, super) martingale. Even if individual discount rates differ both from the interest rate and across individuals, the proposition that anticipated movements in income do not lead to anticipated movements in consumption remains valid. When the new and disappearing consumers are taken into account, this proposition and the martingale characterization are only approximations.

For *investment*, the point was made in connection with the "q theory" of investment. This theory assumes internal costs of adjustment for capital and derives a relation between investment and the ratio of the market value of the firm to its replacement cost. It has the following structure: Assuming, for example, risk-neutral shareholders, a constant interest rate r, a depreciation rate δ, and quadratic costs of adjustment, value maximization implies the following behavior:

$$(2)\qquad \frac{I_t}{K_t}=\delta+\phi(q_t-1)$$

$$(3)\qquad q_t=\sum_{j=0}^{\infty}\left(\frac{1+r}{1-\delta}\right)^{-j}E(MR_{t+j}|\Omega_t)$$

where MR_{t+j} is the marginal revenue from a unit of capital in the firm at time $t+j$. (The exact characterization depends on particular assumptions such as whether capital installed is instantaneously productive and so forth. A closely related derivation is given by Andrew Abel.) The characterization is intuitive: the rate of investment is a linear function of a shadow price q; this shadow price is the present discounted value of expected marginal returns to capital.

Being a present value, q_t satisfies the following relation which is implied by (3):

$$(4)\qquad E\left(q_{t+1}-\left(\frac{1+r}{1-\delta}\right)(q_t-MR_t)|\Omega_t\right)=0$$

This is a familiar relation for asset prices, usually referred to as an arbitrage or no excess return relation. A similar relation with dividends instead of MR_t holds for stock prices for example. (Indeed, under further assumptions, such as a *CRTS* technology, competitive factor and product markets, q is equal to the price of a share, i.e., the title to one unit of capital as valued in the stock market.)

Equation (4) implies that $X_{t+1} \equiv q_{t+1} - ((1+r)/(1-\delta))(q_t - MR_t)$ is a fair game with respect to Ω_t but not that q_t itself follows a martingale. It has been suggested however that q_t follows approximately a martingale. If this is the case, equation (2) implies that the rate of investment follows also approximately a martingale.

To be sure, the above theories have fairly restrictive—and mutually inconsistent (risk-averse consumers but risk-neutral shareholders for example)—assumptions. The intuition behind equations (1) to (4) suggest however that more complex specifications, such as better treatments of uncertainty, are unlikely to yield drastically different implications. They therefore suggest that given interest rates, consumption and investment movements are likely to be mostly unanticipated.

III. Anticipated Movements in Investment

Intuition suggests that the martingale "approximation" is simply wrong for investment: If costs of adjustment are nearly linear, the firm will adjust its capital stock mainly to current demand conditions; if movements in demand are partly anticipated and partly unanticipated, we would expect both large anticipated and unanticipated movements in investment. If, on the other hand, adjustment costs are very convex, the firm will change its capital smoothly. We would then expect both small anticipated and unanticipated movements in investment. In neither case would we expect the ratio of anticipated movements to unanticipated movements to be necessarily small, as required by the martingale approximation.

To see this more clearly, we can solve equations (2) and (3) for two different values of the convexity parameter. The coefficient ϕ in (2) is directly related to this parameter: the more convex adjustment costs are, the lower ϕ. A value of $\phi=.05$ implies that a rate of net investment of 5 percent annually entails a marginal installation cost of 100 percent of purchase price per unit installed. This may reasonably be taken as an upper bound on the convexity of adjustment costs. A value of $\phi=.5$ implies that a rate of net investment of 5 percent entails a marginal cost of 10 percent per unit. This may be taken as a lower bound. Further assume in both cases that the production function is Cobb Douglas, with a share of capital of 25 percent, the depreciation rate is 12 percent annually, the real interest rate is 3 percent, that firms take output as given and that the wage always equals the marginal product of labor. These heroic assumptions allow us to derive the following equation for annual net investment, IN_t:

$$(\phi=.5)\quad IN_t = .20\left[.503\sum_{0}^{\infty}(.69)^i E(Y_{t+i}|\Omega_t) - K_{t-1}\right]$$

$$(\phi=.05)\quad IN_t = .04\left[.290\sum_{0}^{\infty}(.82)^i E(Y_{t+i}|\Omega_t) - K_{t-1}\right]$$

The first term in brackets can be thought of as the desired capital stock. Higher adjustment costs ($\phi=.05$) imply that more weight is given to the distant expected future. They also imply a slower adjustment to the desired stock. If we further assume for example that output is equal to a constant plus white noise ε_t, we get

$$(\phi=.5)\quad IN_t = .80 IN_{t-1} + .1(\varepsilon_t - \varepsilon_{t-1})$$

$$(\phi=.05)\quad IN_t = .96 IN_{t-1} + .01(\varepsilon_t - \varepsilon_{t-1})$$

Higher adjustment costs lead to higher serial correlation, i.e., smaller anticipated movements but also to smaller unanticipated movements. To summarize, investment does not follow a martingale. For plausible values of ϕ ($\phi=.5$ for example), the traditional accelerator theory—with a modified definition of the desired capital stock—still holds and there can be substantial anticipated movements in investment.

What is the empirical evidence on ϕ? Equations such as (2) have recently been estimated and their results are puzzling: they yield implausibly low values of ϕ, usually around .05. Such values of ϕ imply, as we have seen, very large adjustment costs and very small anticipated or unanticipated movements in investment; this is hard to reconcile with the actual movements of investment. There are reasons to believe that these estimates of ϕ are biased downwards: the shadow price q is usually approximated in the regressions by the ratio of market value to replacement cost which is likely to be a mediocre proxy. The market value itself is also surprisingly volatile during the sample period. (This is a puzzle in itself; see Robert Shiller, 1981.) Both reasons would lead to a downwards bias in $\hat{\phi}$.

If investment does not follow a martingale, where did the martingale "approximation" argument of the previous section go wrong? It went wrong in assuming that the fair game property of X_t implies that the present discounted value q_t follows, even approximately, a martingale. Present discounted values do not in general follow martingales; the present discounted value of an $AR(1)$ variable for example follows also an $AR(1)$ with the same coefficient of serial correlation. This was emphasized by Shiller (1979, Appendix A), but the mistake is still quite frequently made.

IV. Anticipated Movements in Consumption

The story is different for consumption. Under the life cycle hypothesis and the additional assumptions made in Section I, individual consumption indeed follows a martingale. Is it true however of aggregate consumption? Can we really disregard the effects of the change in the consumption pool? Suppose, for example, that aggregate income is deterministic and grows at rate γ. Suppose also that $r=\delta$ so that the consumption of each individual is constant. If we look at the total consumption of the agents present in two successive periods, it is constant. Aggregate consumption however grows at rate γ: disregarding the change in the consumption pool is clearly not innocuous. If aggregate income has also a stochastic component, then to the extent that the expected consumption of those who start consuming is different from the consumption of those who stop, there will be some anticipated change in consumption.

To see whether this anticipated change can be large, consider a world in which one agent is born each period and lives for N periods, in which $r=\delta=0$ and in which aggregate income follows $Y_t=a+\rho Y_{t-1}+\varepsilon_t$, each agent receiving $1/N$ of aggregate income. We can derive the behavior of aggregate consumption C_t and look at the ratio of the anticipated change in C_t to its total change by computing

$$A_N=\frac{E(E(C_{t+1}|\Omega_t)-C_t)^2}{E(C_{t+1}-C_t)^2}$$

For $\rho=1$, i.e., if income itself follows a martingale, then $A_N=0$: aggregate consumption also follows a martingale. If on the other hand, $\rho=0$, then A_N is given by

$$A_N=\sum_1^N\left(\frac{1}{n}\right)^2 \Big/ \left[\sum_1^N\left(\frac{1}{n}\right)^2+\left(\sum_1^N\frac{1}{n}\right)^2\right]$$

As the sum in the numerator converges but the second sum in the denominator diverges, A_N tends to zero as N gets large. This implies that the martingale approximation is correct for large N. If we assume for example that agents consume for 50 years, we get $A_{50}=7$ percent and the martingale approximation is quite good. To reject the martingale result, we therefore have to reject some of the assumptions made in Section I.

An obvious candidate is the implicit assumption that wealth can be negative. What happens if wealth cannot be negative for an individual, if there are liquidity constraints? Consider an individual for whom $\delta=r$, so that in the absence of liquidity constraints, his consumption follows a martingale. How will he plan consumption if he expects to be liquidity constrained? He will still never anticipate to decrease his consumption: if he did, there would be a path of constant consumption involving saving now and dissaving later which would yield higher utility and be feasible. He may, however, anticipate to increase his consumption if he anticipates his income to increase: he cannot borrow against future income. Liquidity constraints therefore imply that consumption may not follow a martingale but do not imply that anticipated decreases in income lead to anticipated movements in consumption.

Recent empirical evidence suggests the existence of liquidity constraints. Marjorie Flavin finds that the effect of current income on consumption is too strong to be consistent with the life cycle hypothesis. Robert Hall and Frederic Mishkin, using micro data, conclude that liquidity constraints may affect approximately 20 percent of consumers. This suggests that, although liquidity constraints exist, they may not be so prevalent so as to lead to large anticipated increases in consumption in response to increases in income.

V. Conclusion

Boldly stated, the conclusions of this paper are that anticipated movements in output—especially anticipated decreases—will not lead to changes in consumption but may lead to large changes in investment. In this sense, the multiplier is dead and the accelerator alive.

Given these conclusions, can we, for example, generate a hump-shaped output only from the dynamics of private spending in response to disturbances in autonomous spending? In response to a disturbance, consumption will react and adjust to a new constant level; the anticipated movements in private spending must therefore come from investment.

If disturbances are unanticipated, their occurrence will lead to an increase in consumption and an increase in investment. Over time, consumption will remain approximately constant and investment will decline as the capital stock adjusts. The overall response of output will not be hump shaped, but declining over time.

If disturbances are anticipated, however, say two quarters in advance, they will lead to an initial jump in consumption, an anticipatory increase in investment. After they have occurred, investment will slowly decline. Overall, anticipated white noise disturbances can generate a hump shape of output. Whether the hump shape of output is actually explained by such anticipated disturbances is a different question that this paper does not address, much less answer.

REFERENCES

A. Abel, "Investment and the Value of Capital," New York: Garland Publishing, Inc., 1979.

C. Corrado, "The Steady State and Stability Properties of the MPS Model," unpublished doctoral thesis, Univ. Pennsylvania 1976.

M. Flavin, "The Adjustment of Consumption to Changing Expectations about Future Income," *J. Polit. Econ.*, forthcoming.

R. Hall, "Stochastic Implications of the Life Cycle Permanent Income Hypothesis: Theory and Evidence," *J. Polit. Econ.*, Dec. 1978, *86*, 971–87.

______ **and F. Mishkin,** "The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households," mimeo., July 1980.

P. Samuelson, "Interactions Between the Multiplier Analysis and the Principle of Acceleration," *Rev. Econ. Statist.*, May 1939, *21*, 75–78.

R. Shiller, "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?," *Amer. Econ. Rev.*, forthcoming.

______, "The Volatility of Long Term Interest Rates and Expectations Models of the Term Structure," *J. Polit. Econ.*, Dec. 1979, *87*, 1190–229.