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MENU COSTS AND THE NEUTRALITY OF MONEY*

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A model of endogenous price adjustment under money growth is presented. Firms follow (s,S) pricing policies, and price revisions are imperfectly synchronized. In the aggregate, price stickiness disappears, and money is neutral. The connection between firm price adjustment and relative price variability in the presence of monetary growth is also investigated. The results contrast with those obtained in models with exogenous fixed timing of price adjustment.

I. INTRODUCTION

Historically determined nominal prices can lead to inertia in the aggregate level of prices, leaving room for monetary shocks to influence real variables. Formal models connecting the microeconomic behavior of nominal prices with aggregate price stickiness include models with staggered price and wage decisions [Fischer, 1977; Taylor, 1980; Blanchard, 1983; Parkin, 1986], models with partial adjustment of prices (e.g., Rotemberg [1982]), and the more recent "menu cost" models of Akerlof and Yellen [1985], Blanchard and Kiyotaki [1985], and Mankiw [1985]. We present an alternative aggregate model with microeconomic price stickiness that emphasizes the importance of endogenous timing of price adjustments. The model provides conditions under which money shocks have no real effects.

A number of macroeconomic models of price stickiness have a common microeconomic base: infrequent but large changes in

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nominal variables are assumed to be more economical than frequent small changes.[1] The models also share the assumption that the time between successive price revisions is preset, and hence unresponsive to shocks to the economy. This assumption is questionable both at the microeconomic level and in the aggregate. Formal microeconomic models (e.g., Sheshinski and Weiss [1983]) strongly suggest that more rapid inflation will shorten the time between price revisions. Empirical evidence against the fixed timing assumption is presented by Cecchetti [1986] and Liebermann and Zilbefarb [1985]. At the aggregate level large monetary shocks may increase the number of agents revising their nominal prices in a given period. This in turn reduces the extent of price level inertia. An important open question remains: what are the real effects of monetary shocks with endogenous timing of price revisions?

The present paper assumes that individual firms adjust their prices using (s,S) pricing policies of Sheshinski and Weiss [1977, 1983]. To model asynchronization, we make a cross-sectional assumption on initial prices. The price level is derived endogenously by aggregating across firms. Aggregate price stickiness then vanishes despite the presence of nominal price rigidity and imperfectly synchronized price revisions.

The presence of *relative price variability* as a consequence of inflation is also observed endogenously through aggregation of cross-sectional price data. A simple formula is derived linking nominal price adjustment by firms with cross-sectional variability of inflation rates.

The basic model is outlined in Section II. The neutrality proposition is presented in Section III. In Section IV the model is applied to study relative price variability. Section V provides further discussion of the model and its assumptions. Conclusions are given in Section VI.

II. The Model

IIA. The Aggregate Setting

We provide an aggregate model of price dynamics with individual firms pursuing asynchronous (s,S) pricing policies. The structure of the aggregate model is kept as simple as possible to highlight the distinction between our model and others with asynchronous

1. An exception is Rotemberg [1983] who considers instead increasing marginal costs of nominal price revisions.

price and wage decisions. These alternative models frequently assume a staggered pattern of timing (e.g., Akerlof [1969], Fischer [1977], Taylor [1980], and Blanchard [1983]).

Money growth is subject to continuous shocks. The stochastic process governing monetary growth is taken as exogenous by all firms in the economy.[2] Let $M(t)$ denote the logarithm of the money supply at time t, where time is measured continuously. We assume that the money supply process is increasing over time and does not make discrete jumps.

ASSUMPTION 1. *Monotonicity and Continuity.* The money supply does not decrease over time, $M(t_2) \geq M(t_1)$ for $t_2 \geq t_1$. Also, the money supply process is continuous in the time parameter t. Normalize such that $M(0) = 0$.

The monotonicity assumption will rule out periods of deflation. The continuity assumption allows a simple characterization of firm pricing policies. The assumption also plays a role in analyzing the cross-sectional behavior of prices. This issue is taken up below. The monetary process is sufficiently general as to accommodate feedback rules. We shall consider particular examples of monetary processes below.

There is a continuum of firms in the economy indexed by $i \in [0,1]$. All firms face identical demand and cost conditions. The assumed microeconomic structure is based on the menu cost model of Sheshinski and Weiss [1977, 1983]. Let $q_i(t)$ and $Q(t)$ represent firm i's *nominal price* and the *aggregate price index,* respectively, with $p_i(t)$ and $P(t)$ their respective logarithms. The aggregate price index, $P(t)$, is derived endogenously below from individual firm prices. It is convenient to express firm i's *real price,* $q(t)/Q(t)$, in log form, $r_i(t)$,

$$r_i(t) \equiv p_i(t) - P(t) = \ln\,[q_i(t)/Q(t)], \tag{1}$$

for all $i \in [0,1]$. We take $r_i(0)$ as given.

The *aggregate price index* $Q(t)$ is determined endogenously by aggregating individual firms' nominal prices $q_i(t)$. The index is assumed to depend only on the frequency distribution over nominal prices. Because firms have menu costs of price adjustment, prices may remain dispersed in the long run. Thus, the set of observed prices at any date may be described by a time-dependent frequency distribution function, say $G_t(q)$. The index is assumed also to

2. In general, the money growth process may be set as a feedback rule based on the history of output.

satisfy homogeneity; when nominal prices double, so does the index.[3]

ASSUMPTION 2. *Symmetric Price Index.* The aggregate price index $Q(t)$ depends only on the frequency distribution of nominal prices and satisfies homogeneity:

$$Q(t) = Q(G_t(q)), \text{ where } G_t(q) \text{ is the proportion of firms } i \in [0,1] \text{ such that } q_i(t) \leq q, \tag{2}$$

$$\text{if } G_{t_1}(q) = G_{t_2}(\lambda q) \text{ for all } q, \text{ then } \lambda Q(t_1) = Q(t_2), \text{ for any } t_1, t_2 \geq 0. \tag{3}$$

This condition is satisfied by a wide variety of common price indices.[4] An example of a price index that satisfies Assumption 2 is a simple *average* of nominal prices based on their frequency distribution, $Q(t) = \int q dG_t(q)$. More generally, let $Q(t) = \int w(q,G_t(\cdot))q dG_t(q)$, where $w(q,G)$ represents weights as a function of prices q and the distribution of nominal prices G. The assumption requires the weights to satisfy $w(q,G_{t_1}) = w(\lambda q, G_{t_2})$ when $G_{t_1}(q) = G_{t_2}(\lambda q)$ for all q. An example of such a set of weights is $w(q,G) = q/\int q dG(q)$.

IIB. The Market Setting

Consumer demand is assumed to depend only on the firm's real price and on real money balances. Writing the arguments in log form, *consumer demand* faced by firm i, Γ_i, is defined by

$$\Gamma_i(t) \equiv \Gamma(r_i(t), M(t) - P(t)), \tag{4}$$

where $r_i(t)$ and $M(t) - P(t)$ are the log of firm i's price and the log of real balances, respectively.[5] One rationale for this is to assume that real balances enter consumer utility functions, as in, for example, Rotemberg [1982, 1983]. Note also that all firms can have

3. Individual firms set s and S taking the price level as exogenously given. However, for given levels s and S, the index endogenously determines $P(0)$: will the exogenous and endogenous indices be consistent? The answer is generally no: however, if we associate higher real balances with higher levels of s and S, there will be some initial specification of real balances guaranteeing this static consistency, since higher real balances raise the desired average real price, raising the endogenous level of $P(0)$ relative to the exogenous level.

4. Blanchard and Kiyotaki [1985] and Ball and Romer [1986] derive symmetric price indices based on an underlying symmetric utility framework.

5. The assumption that demand is independent of future prices rules out consumer speculation. Benabou [1985a] presents an analysis of optimal pricing policies in the face of consumer storage and speculation. In principle, the future path of real money balances may also influence real demand. For present purposes, Proposition 1 will allow us to ignore this potentially complex dependence.

some positive demand even though prices are dispersed. This may arise if the commodities are imperfect substitutes. It may also be that consumer search across firms is costly and that consumers do not recall prices posted by firms in earlier periods (see Benabou [1985b]).

Costs are assumed to be fixed in real terms. Production at rate $X_i(t)$ gives rise to real flow costs, $C(X_i(t))$. This assumption rules out stickiness in nominal input prices, including contractual wages. This prevents us from addressing the relationship between price stickiness and wage stickiness, a topic of independent interest (see Blanchard [1983]).[6] Additional study of the present model with input price stickiness is clearly desirable. All profits are distributed to consumers, and firm costs accrue to consumers as income.[7]

The good is assumed to be nonstorable, so that the firm's output is supplied at the same date it is produced. This removes intertemporal linkages embodied in inventories. As a result, the only variables that influence the firm's *flow rate of real profits* $B_i(t)$ are the instantaneous real price and the level of real money balances:[8]

$$B_i(t) \equiv B[r_i(t), M(t) - P(t)] = \max_{X_i(t) \leq \Gamma_i(t)} [e^{r_i(t)} X_i(t) - C(X_i(t))]. \tag{5}$$

Thus, the *output* of firm i, $X_i(t)$, is a function of its real price and the level of real money balances which solves the problem in equation (5):

$$X_i(t) = X(r_i(t), M(t) - P(t)). \tag{6}$$

Let $X(t)$ represent the constant dollar value of *aggregate output*:

$$X(t) \equiv \int_0^1 (q_i(t)/Q(t)) X_i(t) di = \int_0^1 e^{r_i(t)} X_i(t) di.$$

In the absence of menu costs, the firm picks its instantaneous price $r_i(t)$ to maximize flow profits $B(r_i(t), M(t) - P(t))$.[9] Nominal price stickiness is introduced into the model in the form of a real

6. Gordon [1981] finds evidence for price stickiness for periods with widely different forms of labor contract. This suggests that there are important sources of price stickiness other than the behavior of input prices.

7. By Walras' law, market clearing in the commodity market implies market clearing in the money market; see, for example, Rotemberg [1982].

8. The present formulation allows the firm to ration its customers. The case without rationing can also be handled by the model; see Sheshinski and Weiss [1983].

9. With standard assumptions, increases in real money balances that increase demand for the commodity will also raise the firm's optimal real price.

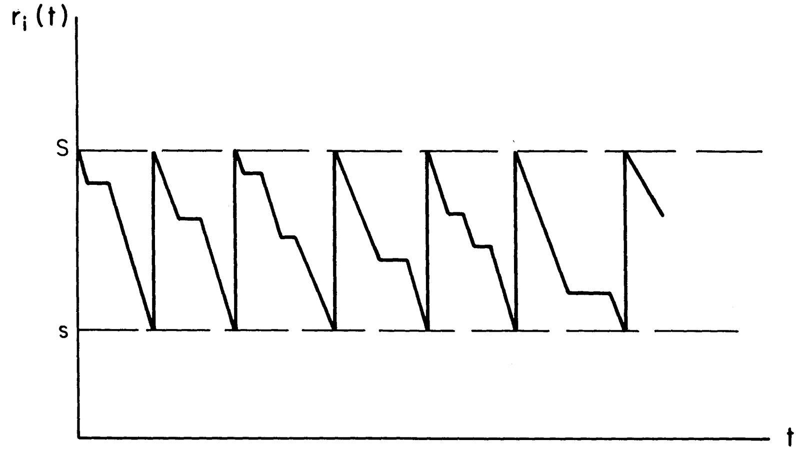

FIGURE I

menu cost, β, which is incurred each time the firm changes its nominal price.[10] This fixed transaction cost results in price stickiness at the level of the individual firm. Rather than responding smoothly and continuously to changes in the overall price level the firm responds only occasionally, and with discrete price jumps.

We consider a firm that continuously monitors the price level, and pursues an (s,S) *pricing policy,* as introduced by Sheshinski and Weiss. The impact of this policy on the dynamics of the firm's real price is illustrated in Figure I. The instant the log of the real price $r(t)$ hits the fixed lower limit s, the firm adjusts its nominal price, returning the log of the real price to its upper limit S. Let $D \equiv S - s$ represent the size of the firm's price increase. Then, the changes in the firm's nominal price within any time period $[0,t]$ are always an integer multiple of the price range, $p(t) - p(0) = k(t)D$, where $k(t) \geq 0$ is an integer. Noting that $r_i(0) = p_i(0)$ and using the definition of the firm's real price in equation (1), we may formally characterize the (s,S) pricing policy as follows: $r_i(t) \in (s,S]$ and

$$r_i(t) - r_i(0) = (p_i(t) - p_i(0)) - (P(t) - P(0)) \tag{7}$$
$$= k_i(t)D - (P(t) - P(0)).$$

10. There is an issue here concerning the proper treatment of menu costs. If these are indeed real costs, they should be explicitly included as part of output. Hence a closed model of the economy should properly include a sector of variable size dedicated to the production of menus. This is ignored in our formulation.

Hence, changes in the log of the firm's real price are an integer multiple of D minus the log of the price level.

Two important requirements are necessary for (s,S)-type policies to be optimal. One requirement is stationarity of real balances over time—$M(t) - P(t) = -P(0)$, so that demand Γ_i is stationary. We shall demonstrate that in equilibrium this requirement is satisfied. The other requirement concerns restrictions on the form of the anticipated inflation process. Conditions for optimality of (s, S) pricing policies in a stochastic setting have been considered by Sheshinski and Weiss [1983], Danziger [1984], and more recently by Caplin and Sheshinski [1987].[11] Danziger considers a world with discrete inflationary shocks. He demonstrates that when inflationary shocks arrive one at a time with exponentially distributed interarrival times, then the optimal pricing policy is of the (s,S) variety.[12] With general inflationary processes, the optimal pricing policy may take a more complex form.

The central qualitative feature of (s,S) pricing policies is that they make the time between successive price revisions endogenous: prices change more frequently when inflation is rapid than when it is slow. Alternative models of asynchronous price setting involve fixed decision times regardless of ensuing shocks to the economy. Seen in this light, one may be less concerned with the precise optimality of (s,S) pricing policies.[13] Rather, they may be seen as a simple and tractable alternative to the assumption of a predetermined pattern of price revisions.

Analysis of the time path of aggregate prices in our framework requires specification of the initial distribution of prices across firms in the economy. It is assumed that firms' initial real prices $r_i(0)$ are uniformly distributed over the range $(s,S]$. For ease of exposition we restate the uniformity assumption with a frequency distribution $F_0(p)$ which defines the proportion of firms with the logs of their initial prices $p_i(0)$ no higher than p.

11. Sheshinski and Weiss [1983] employ a special form of the stochastic inflation process. Caplin and Sheshinski [1987] present a discrete time formulation with i.i.d. inflationary shocks.

12. While the discrete nature of Danziger's inflation process contradicts Assumption 1, our analysis including the neutrality proposition nevertheless applies.

13. Even in the inventory literature, Arrow, Harris, and Marschak [1951] study (s,S) policies because of their relative simplicity. The first general proof of optimality is due to Scarf [1959]. Further, stationary (s,S) policies are frequently analyzed and applied in situations where they are undoubtedly suboptimal (such as in multi-echelon inventory systems [Schwarz, 1981] and in more general nonstationary environments [Karlin and Fabens, 1959].

ASSUMPTION 3. *Uniformity*. The frequency distribution over initial real prices satisfies

$$F_0(p) = \begin{cases} 0 & \text{for } p \le s, \\ b/D & \text{for } p = s + b, \text{ with } 0 \le b \le D, \\ 1 & \text{for } p \ge S. \end{cases} \tag{8}$$

The uniform initial distribution of prices across the price range $(s,S]$ is the analogue in prices of the standard assumption of uniformly staggered price changes over time. Indeed, Assumption 3 is equivalent to an assumption of uniform staggered timing in the special case where inflation is constant at some rate $\lambda > 0$. However, it will be apparent that in a stochastic setting a uniform distribution of initial prices has significantly different implications.

In a fundamental sense Assumption 3 may be viewed as a statement about the endogenous tendency of prices to become uniformly distributed after a long history of inflationary shocks and pursuit of fixed (s,S) policies. This lies outside the current framework, since firms pursuing identical (s,S) policies in the face of inflation retain forever the initial difference in their real prices. However, if firms pursue slightly distinct (s,S) policies, or randomize on their trigger price s (as in Benabou [1985a]), their real prices become statistically independent of one another with the passage of time. A related result for inventories states that, absent degeneracies, firms that pursue (s,S) inventory policies have inventory levels that are independent in the long run [Caplin, 1985].

III. NEUTRALITY

We address the connection between asynchronous price decisions and aggregate price stickiness. To what extent is the individual firm stickiness in nominal prices reflected in aggregate price inertia? The central result of the paper is that *real balances and aggregate output are invariant to monetary shocks*. Price stickiness disappears in the aggregate. Given (s,S) pricing rules, the initial distribution of real prices is invariant and remains uniform. The aggregate nominal price index exactly reflects nominal money shocks. Consumer demand as a function of real prices and real balances remains stationary. This results in constant aggregate output.

In the absence of real shocks to the economy, money neutrality is appropriately defined as follows.

DEFINITION 1. Money is *neutral* if aggregate real output is invariant to monetary shocks, $X(t) = X(0)$, for all $t \geq 0$.

Monetary policy may influence the *distribution* of real prices across firms in our model as will be seen in Section IV. However, these distributional effects cancel out in the aggregate.

Suppose that firms follow (s,S) policies in anticipation of constant real balances. That is, firms expect that $P(t) = M(t)$. Then, by the description of (s,S) pricing policies in equation (7), we may calculate each firm's nominal price as a function of cumulative money growth and the firm's initial price:

$$p_i(t) = k_i(t)D + p_i(0), \tag{9}$$

where $k_i(t)$ is an integer determined by the requirement that $r_i(t) \in (s,S]$. Proposition 1 verifies that aggregation of these nominal prices yields a price level equal to cumulative money growth at each time t, so that money is neutral.

The neutrality result may be understood by observing that the (s,S) policy moves real prices around a circle. The method of proof

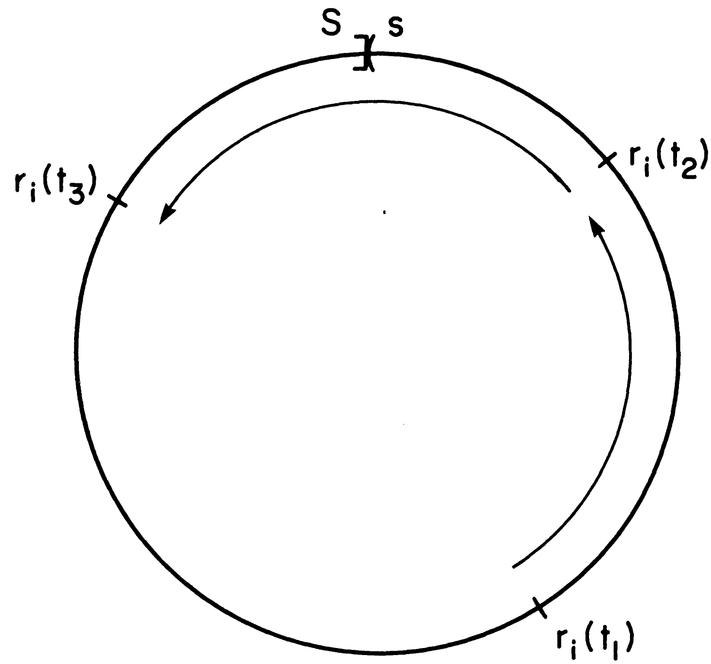

FIGURE II

is easily illustrated using Figure II. Points on the circle represent the range of the log of the firm's real prices. At the apex of the circle, the outer limits of the range are adjacent. At time t_1, $r_i(t_1)$ is firm i's real price. Inflation occurring between time t_1 and t_2 reduces the real price to $r_i(t_2)$ as indicated by the counterclockwise motion. Between time t_2 and t_3, inflation drives the real price down to s, the price is then readjusted up to S and further inflation drives the real price to $r_i(t_3)$. It is critical to note that the rotation engendered by monetary growth is invariant to the location of the initial real price on the circle, thus preserving the initial uniformity of real prices.

PROPOSITION 1. Given Assumptions 1 to 3, money is neutral if firms follow (s,S) pricing policies in anticipation of constant real balances.

Proof of Proposition 1. Let money growth be written as an integer multiple of D and a remainder $b(t)$:

$$M(t) = k(t)D + b(t), \tag{10}$$

where $k(t) \geq 0$ and $b(t) \geq 0$ are chosen such that $b(t) < D$. If firms follow (s,S) pricing policies and anticipate constant real balances, then by equation (9), the log of each firm i's nominal price can be expressed in terms of the components of money supply growth in equation (10):

$$p_i(t) = \begin{cases} p_i(0) + k(t)D, & \text{for } p_i(0) > s + b(t), \\ p_i(0) + [k(t) + 1]D, & \text{for } p_i(0) \leq s + b(t). \end{cases} \tag{11}$$

Equation (11) shows that if $s + b(t) < p_i(0) \leq S$, then $s + M(t) < p_i(t) \leq s + M(t) + D - b$. Also if $s < p_i(0) \leq s + b(t)$, then $s + M(t) + D - b < p_i(t) \leq S + M(t)$. By uniformity of initial real prices (Assumption 3), it follows that $p_i(t) - M(t)$ is uniform over the interval $(s,S]$, or equivalently,

$$F_t(p) = \begin{cases} 0 & \text{for } p \leq s + M(t), \\ b/D & \text{for } p = s + M(t) + b, \text{with } 0 \leq b \leq D, \\ 1 & \text{for } p \geq S + M(t). \end{cases} \tag{12}$$

The frequency distribution over nominal prices is then given by $G_t(q) \equiv F_t(\ln q)$. Note that $G_t(q)$ is defined over $(e^{s+M(t)},e^{S+M(t)}]$. Thus, we may define $G_t(e^{M(t)}x)$ over (e^s,e^S) so that $G_t(e^{M(t)}x) = G_0(x)$ for $x \in (e^s,e^S)$. Therefore, by the assumption of a symmetric price index, $Q(t) = e^{M(t)}Q(0)$. Thus, we have verified that endogenously derived inflation matches monetary growth and real balances are constant: $Q(t)/e^{M(t)} = Q(0)$. Furthermore, since $r_i(t) = p_i(t) - P(t= p_i(t) - M(t)$ is uniform over $(s,S]$ for $t \geq 0$, we have

$$(13)\quad X(t) = \int_0^1 e^{r_i}(t)X(r_i(t), P(0))di = \int_0^1 e^{r_i(0)}X(r_i(0), P(0))di,$$

so $X(t) = X(0)$.

Q.E.D.

Consider an illustrative example. Note first that since $p_i(t)$ is uniformly distributed on $(s + M(t), S + M(t))$, $q_i(t)$ is distributed on $(e^{sM(t)}, e^{SM(t)})$ with distribution $G_t(q) = (\ln q)/D$. If we use the simple arithmetic mean as our symmetric price index, the price level is then

$$Q(t) \equiv \left(\frac{1}{D}\right) \int_{e^{sM(t)}}^{e^{SM(t)}} dq = e^{M(t)} \frac{e^S - e^s}{D} = e^{M(t)}Q(0).$$

The central feature of Proposition 1 is that it provides a simple framework in which there are monetary shocks, asynchronous nominal price revisions, but no stickiness in the aggregate price level. In fact, $P(t) - M(t) = P(0)$. Thus it contrasts strongly with monetary models with a fixed staggered pattern of price and wage revisions, which can generate significant aggregate price stickiness (e.g., Akerlof [1969], Blanchard [1983], and Fischer [1977]). In qualitative terms, the difference between the results can be simply explained. In the staggered timing framework, large monetary shocks draw a response from a fixed fraction of the population, with the remainder pursuing an unchanged policy. The size of the predetermined pool of decision makers will influence the extent of price revision by those currently free to decide: on average, agents' prices adjust only partially to large monetary shocks. In contrast, the (s,S) model makes the fraction of firms that revise prices in any given period endogenous. Hence rapid growth of the money supply causes an increase in the number of price increases in a given period. Surprisingly, our simple form of endogenous timing completely removes aggregate inertia.

The result also provides a new perspective on the emerging study of menu costs and monetary policy in a *static* setting (e.g., Akerlof and Yellen [1985], Blanchard and Kiyotaki [1985], Mankiw [1985]). Here, Akerlof and Yellen [1985] argue that the presence of a small menu cost may make it optimal for an individual firm to maintain a fixed nominal price in the face of a monetary shock. This may lead to a welfare loss larger than the menu cost itself. The extension from the case of a single firm to the economy as a whole is based on a representative agent framework. Since one firm fails to adjust its price, so do all firms, and as a result the open market operation can have a significant real impact.

Taken literally, such reasoning can only be applied for the first monetary shock to an economy that had never before been out of static equilibrium. Even the second monetary shock may have a different effect, since after the first shock, the hypothesis that the initial real price is at its equilibrium level fails. Proposition 1 presents a simple setting where the presence of menu costs indeed prevents many firms from revising prices. However, those who do adjust their price do so discontinuously. Although only a few firms may adjust their prices, they adjust their prices by a large amount. The net result is that monetary shocks are absorbed with no real impact.

Proposition 1 also provides a positive answer to a question posed by Sheshinski and Weiss [1983] for their model of (s,S) pricing policies. They are concerned with providing a consistent aggregate version of their model. They consider identical firms facing exogenous inflationary shocks, uniformly distributed with respect to the time of their last price increase. Sheshinski and Weiss [1983, p. 523] note that:

> large and/or closely spaced shocks may lead to synchronization and hence change the distribution. There is thus no simple correspondence between the process of exogenous shocks and the process followed by the aggregate price level.

Proposition 1 demonstrates that with identical firms, consistent aggregation requires that firms be uniformly distributed in terms of the log of their *initial real price* levels rather than the time of their last price change. The distinction is that in a stochastic setting uniformity in timing is unstable, while uniformity in real prices is continuously sustained.[14]

IV. Menu Costs and Relative Price Variability

In this section we develop formulae linking inflation and firm pricing policies to relative price variability. These formulae can be seen as stochastic generalizations of the deterministic price dispersion models of Rotemberg [1981], and Cecchetti [1985], which are based on staggered price setting. Our results also clarify the

14. In a deterministic world with constant inflation, the two forms of uniformity are equivalent.

relationship between price variability and the time period between successive observations of the economy.[15]

The association between inflation and relative price variability has been widely investigated; see Fischer [1981] for a survey. The empirical research suggests a positive association between relative price variability and both the mean and the variance of the overall rate of inflation.[16] One important line of research into inflation and relative prices originates with Barro [1976]. Here it is inflationary variability rather than the rate of inflation per se that drives relative price variability. As the variability of inflation increases, individual firm estimates of inflation become more widely dispersed, driving apart firms' preset prices.[17] Barro's approach is further developed by Cukierman [1979], Cukierman and Wachtel [1982], Hercowitz [1981], and Parks [1978].

An alternative theory holds that inflationary variations in relative prices can be caused by nominal price inflexibility [Cecchetti, 1985; Mussa, 1981; Rotemberg, 1983].[18] Our formulae lie in this alternative tradition, stressing the costs of changing nominal prices.

The basic characterization of relative price variability to be given here is based on repeated observations of the economy, with successive observations separated by a fixed time period of arbitrary length $\tau > 0$. With this discrete pattern of observations, cumulative inflation during the tth time period is denoted $\Pi^{\tau}(t)$. Proposition 1 allows us to identify the inflation rate with the (stochastic) growth of the money supply:

$$(14) \qquad \Pi^{\tau}(t) \equiv P[\tau(t+1)] - P[\tau t] = M[\tau(t+1)] - M[\tau t].$$

Our results of this section require only that $\Pi^{\tau}(t)$ is a stationary stochastic process. It is also convenient to restrict attention to inflation or money supply processes that are regularly behaved.

15. As Cecchetti [1985] notes in a nonstochastic setting, there is no cross-sectional variance of inflation rates when the observation period is an integer multiple of the period between price revisions.

16. Early studies include Graham [1930] and Mills [1927]. More recent work includes Vining and Elwertowski [1976]; Pagan, Hall, and Trivedi [1983]; Balk [1985], and Marquez and Vining [1984].

17. According to this approach, the apparent association between the level of inflation and relative price variability is a statistical artifact, resulting from an actual association between the mean level of inflation and the variability of inflation. This relationship is explicitly investigated by Taylor [1981].

18. See also Carlton [1978] and Hubbard and Weiner [1985], who consider markets with both spot transactions and nominal contracting.

ASSUMPTION 1A. *Stationarity.* For any $\tau > 0$, the process $\Pi^{\tau}(t)$ of equation (14) is a stationary stochastic process, with long-run probabilities specified by the density function $\phi^{\tau}(\Pi)$. The density of $\phi^{\tau}(\Pi)$ is assumed to be non-atomic, with compact support.

As in the proof of Proposition 1, it is useful to separate inflation into an integer multiple of D and a residual.[19] Definition 2 provides the appropriate formalization.

DEFINITION 2. With cumulative inflation measured over periods of length $\tau > 0$, the *residual inflation process* $b^{\tau}(t)$ is defined as $\Pi^{\tau}(t)$ taken modulo D.

In light of Assumption 1A, the residual process $b^{\tau}(t)$ is itself stationary and has compact support, with long-run probabilities specified by the density function $\eta^{\tau}(b)$ satisfying,

$$\eta^{\tau}(b) = \sum_{k=0}^{\infty} \phi^{\tau}(kD + b). \tag{15}$$

Individual firm price increases are also measured at intervals of length τ:

$$\Pi_i^{\tau}(t) \equiv p_i[\tau(t+1)] - p_i(\tau t). \tag{16}$$

To measure *inflation,* we use a specific price index. This is the standard *Divisia* index of inflation, with equal expenditure shares for distinct firms $i \in [0,1]$:

$$\Pi^{\tau}(t) \equiv \int_0^1 \Pi_i^{\tau}(t)\, di. \tag{17}$$

The Divisia index is a standard employed in empirical studies of relative price variability (e.g., Fischer [1981], Hercowitz [1981], Parks [1978], and Vining and Elwertowski [1976]). The Divisia index is symmetric. By Proposition 1, it follows that the endogenous inflation measure in equation (17) is consistent with monetary growth in equation (14).

Relative price variability $V^{\tau}(t)$ is measured as the dispersion of individual firm inflation rates around the aggregate rate of inflation:

$$V^{\tau}(t) \equiv \int_0^1 [\Pi_i^{\tau}(t) - \Pi^{\tau}(t)]^2 di. \tag{18}$$

19. The formal identification between (s,S) policies and the modulo arithmetic also plays a role in the inventory literature (see Caplin [1985]).

We are interested in the statistical properties of $V^\tau(t)$, and in particular the influence of D, the size of individual price increases. Intuition suggests that increases in D may raise the general level of relative price variability. A precise characterization of the expected level of relative price variability is contained in Proposition 2.

PROPOSITION 2. Expected relative price variability is related to price changes D and the residual inflation process $b^\tau(t)$, as follows:

$$E[V^\tau(t)] = E\{b^\tau(t)[D - b^\tau(t)]\}, \tag{19}$$

with $b^\tau(t)$ as in Definition 2.

Proof of Proposition 2. To simplify notation, the superscript τ is suppressed throughout the proof. We first separate period t inflation in the standard manner,

$$\Pi(t) = k(t)D + b(t), \tag{20}$$

with $k(t)$ a nonnegative integer, and $0 \leq b(t) < D$. The (s,S) pricing policies imply that individual firm price increases obey

$$\Pi_i(t) = \begin{cases} k(t)D & \text{for } r_i(t) > s + b(t), \\ [k(t) + 1]D & \text{for } r_i(t) \leq s + b(t). \end{cases} \tag{21}$$

Hence $(\Pi_i(t) - \Pi(t))^2$ takes value b^2 for $r_i(t)$ above $s + b(t)$, $(D - b)^2$ otherwise. But from Proposition 1 we know that real prices $r_i(t)$ are distributed uniformly over $(s,S]$ for $t \geq 0$. Hence, using the definition of $V(t)$, we have

$$\begin{aligned} V(t) &= \left(\frac{D - b(t)}{D}\right)b^2(t) + \left(\frac{b(t)}{D}\right)[D - b(t)]^2 \\ &= b(t)(D - b(t)). \end{aligned} \tag{22}$$

Finally, Assumption 1A implies that $b(t)$ is a stationary process, allowing us to take expectations in (22).

Q.E.D.

Proposition 2 shows that the range of individual price variation D is a central determinant of the variability of individual price increases. However, interpretation of the result is complicated by the presence of the residual process, $b^\tau(t)$. While the formula does suggest a positive association between D and relative price variability, examples with a negative association are readily constructed.[20]

20. For example with $\Pi^\tau(t)$ uniform over [9,10] an increase in D from 8 to 9 reduces $EV^\tau(t)$ from $9\frac{2}{3}$ to $4\frac{1}{6}$.

By changing the time interval between observations, it is possible to greatly simplify the formulae of Proposition 2. The results are stated for a restricted class of inflation processes introduced in Assumption 1B.[21] The restriction is imposed to simplify proofs: the analysis may incorporate more general conditions.

ASSUMPTION 1B. *Two-rate inflation process.* Monetary growth (and hence inflation) can take place at one of two distinct rates, g_H and g_L with $g_H > g_L \geq 0$. The time spent with inflation of g_H (respectively, g_L) is distributed exponentially with parameter λ_H (respectively, λ_L).

A desirable feature of the two-rate inflation processes of Assumption 1B is that their simple Markovian structure is inherited by the discretely observed process $\Pi^\tau(t)$. The state of the system at time t comprises a specification of all firms' instantaneous real prices $r_i(t)$, and the current inflation rate, H or L. State transitions in the ensuing interval depend only on cumulative inflation over the interval, and the level of inflation at the end of the interval. Such state transitions are then Markovian, since information available prior to t is irrelevant to the probabilistic progress of the system.[22]

With this background, we can provide the simple formulae of Proposition 3 which apply, respectively, to "widely spaced" and to "closely spaced" observations of the economy. The Proposition is proved in the Appendix.

PROPOSITION 3. Given Assumptions 1B, 2, and 3, if firms follow (s,S) pricing policies and τ is the period of observation, then expected relative price variability satisfies the following:

(a) $$\lim_{\tau \to \infty} EV^\tau(t) = D^2/6,$$

(b) $$\lim_{\tau \to 0} \left[\frac{EV^\tau(t)}{E\Pi^\tau(t)}\right] = D.$$

The surprising feature of part (a) of Proposition 3 is that with widely separated observations, relative price variability depends only on D. It may be that the formula is roughly appropriate for

21. Assumption 1B represents a slightly more general form of the inflation process studied by Sheshinski and Weiss [1983].

22. Note that transitions in the rate of inflation between observations are not independent of cumulative inflation. High cumulative inflation is associated with an ensuing inflation rate of g_H. Hence transition probabilities for the Markov process are nonseparable between real price transitions and transitions in the inflation rate.

semiannual data where firms change prices at intervals ranging from one to three months. The applicability of part (b) of Proposition 3 is harder to gauge: the observation period must be considerably shorter than the time between successive price revisions.

Sheshinski and Weiss [1983] provide useful formulae for assessing the impact of parameter changes on $D = S - s$, the range of the log of real prices.[23] For $g_L = 0$, they establish that the range D is increasing in the price adjustment cost β and increasing in the certainty-equivalent rate of inflation $\bar{g}$, where $\bar{g} = (\lambda_L + \rho)g_H/(\lambda_L + \lambda_H + \rho)$ and where ρ is the rate of interest. Changes in parameter values λ_L, λ_H, ρ, and g_H will affect the price range and thus relative price variability as defined in Proposition 3a. However, it is difficult to establish a direct relation between the mean and variance of inflation and relative price variability.

It is possible to determine the effects of menu costs on relative price variability. Because (s,S) policies may not be optimal, we assume that firms choose the *best* (s,S) bounds. Then, we use a time period $\tau \leq D/g_H$. Since $\Pi(t) = k(t)D + b(t)$ from equation (20), the number of nominal price changes within the time period under observation is always zero so that $\Pi(t) = b(t)$. Then, we may write expected relative price variability, using Proposition 2, as follows:

$$E[V^{\tau}(t)] = E\{\Pi(t)[D - \Pi(t)]\}. \tag{23}$$

The inflation process $\Pi(t)$ is independent of adjustment costs, and the range of prices is increasing in β. Thus, if firms follow the best (s,S) pricing policy, expected relative price variability is increasing in the menu costs of price adjustment β.

V. Interpretation of Assumptions

The neutrality of money in our model is particularly dependent on the (s,S) form of firm pricing policies. For firms to follow (s,S) policies, the monetary process must at least exhibit monotonicity and continuity. These requirements may be quite restrictive.

When the monetary process is nonmonotone, it will sometimes be necessary for the firm to *lower* its nominal price. The one-sided (s,S) pricing policies must be replaced by two-sided pricing policies,

23. The related (s,S) inventory literature suggests that increases in the mean and variance of sales will raise order size. The well-known Wilson lot-size formula (more familiar as the square-root formula for money demand) expresses the relationship in simple form. The more recent approximation formula of Ehrhardt [1979] has similar properties.

as analyzed by Barro [1972].[24] With the two-sided pricing policies, the neutrality proposition no longer holds: it may even be that unusually rapid monetary expansion is associated with increased real balances and vice versa.[25] A theoretical difficulty in modeling two-sided policies is that their properties under aggregation appear highly complex. Specifically, it is not possible to specify an initial cross-sectional distribution of prices which survives shocks.[26] In economic terms, this implies that a second positive shock to the money supply may have very different effects than the first positive shock. Such effects may well have non-intuitive implications: for example, after two successive positive shocks, output may be higher in response to a negative than in response to a third positive shock to the money supply. In the absence of a fully developed model, such comments remain speculative.

The assumed continuity of the money supply process has two roles. First, it gives rise to the simple form of the individual firm equations for price transitions. In particular, (7) no longer holds in the absence of continuity, since if the real price falls by a discrete amount at any given instant, then it may at some point fall strictly below s. The immediate response of increasing the real price to S then involves a discrete jump in the real price in excess of $D \equiv S - s$, contradicting (7). Sample path continuity plays an additional role in relation to the uniformity Assumption 3. Jumps in the price level act as a coordinating device, pulling many firms in the economy to adjust at the same instant, and eliminating uniformity. The uniform distribution over initial prices, however, is the only distribution that is invariant to shocks.

Finally, there are conditions under which alternative pricing policies may be optimal. Significant alterations in the monetary process may lead agents to revise trigger points.[27] One possibility is that a sudden increase in the rate and variability of money growth causes all agents to broaden their trigger range, raising S and lowering s. In this case, real balances may rise in the short run as firms find insufficient benefit from a price change. This increase in real balances corresponds to the effect noted in the literature on the impact of menu costs in a static setting, as in Akerlof and Yellen [1985]. Once again, note that the short-run expansionary impact of

24. An analogous model of money holding with both inflows and outflows is due to Miller and Orr [1966].
25. A suggestive example is presented in Blanchard and Fischer [1985].
26. This will, of course, invalidate the neutrality proposition.
27. Blinder [1981] examines the related issue of changing trigger points and their impact on aggregate inventory behavior.

monetary policy is not stable. When real balances have risen enough, a sudden burst of price increases may be triggered as all firms go to the very top of their real price range. This process will result in a reduction of real balances to below their initial level, and a corresponding slowdown in activity.

The neutrality result depends on firms anticipating constant real money balances. What would happen if firms anticipated systematic changes in real money balances? For example, if firms expect real money balances, and therefore demand, to increase, this may trigger an earlier price increase, thus counteracting the rise in real balances. A formal analysis of this possibility is of interest.

It is worthwhile noting a concern about the exogenous demand functions Γ_i—particularly in evaluating comparative dynamics. It would, of course, be desirable to construct the demand functions endogenously from consumer utility functions with either differentiated products or consumer search. Ball and Romer [1986] derive such demand functions in a general equilibrium model with differentiated products. With endogenous search activity, demand at a real price of $r_i(t) > 0$ may be zero if all other firms have identical prices $r_j(t) = 0$, but positive if other firms have widely dispersed prices. Hence the functions $\Gamma[r_i(t), M(t) - P(t)]$ must be treated as conditional on the levels of S and s in the rest of the economy. Benabou [1985b] provides a thorough treatment of the interaction between search and menu costs.

VI. Conclusion

The paper presents a model in which inflation is derived endogenously through price adjustment by firms. If firms pursue (s,S) price adjustment policies and the log of real prices are initially uniformly dispersed, then money shocks are shown to be neutral. Thus, nominal changes, such as monetary growth, do not have aggregate real effects despite the presence of menu costs of price adjustment. Although money is neutral, we observe the presence of relative price variability.

The model illustrates that individual firm price stickiness and staggered timing need not lead to aggregate price stickiness. This suggests that real effects of money shocks may depend more on fixed-length contracts than simply on asynchronous nominal price adjustment. Overall, the analysis highlights the importance of cross-sectional timing assumptions in macroeconomic models.

APPENDIX

Proof of Proposition 3

To prove part (a) in light of Proposition 2 requires only that

(A1) $$\lim_{\tau\to\infty} \{E[b^\tau(t)(D - b^\tau(t))]\} = D^2/6,$$

with $b^\tau(t)$ as in Definition 2. Let $H^\tau(x)$ denote the long-run cumulative distribution of $b^\tau(t)$:

(A2) $$H^\tau(x) = \int_0^x \eta^\tau(b)\, db.$$

The heart of the proof of part (a) is contained in Lemma 1.

LEMMA 1. For $0 \leq b \leq D$, $\lim_{\tau\to\infty} H^\tau(x) = b/D$.

Proof. With the simple two-level inflation process of Assumption 6, the individual firm's discretely observed real price behavior is ergodic, with a unique stationary density $\psi(r_i(t))$ which is uniform over $(s,S]$. Ergodicity can be proved by applying the procedure of Caplin and Spulber [1985, Proposition 1]. The trivial amendment concerns the fact that g_H and g_L may both be positive in the current case: in the earlier version $g_L = 0$. The existence of this simple ergodic distribution implies that

(A3) $$\lim_{\tau\to\infty} P\{r_i(t + \tau) \in (S - b,X)\,|\,r_i(t) = S\} = b/D.$$

But $r_i(t) \in [s,S]$ and equation (7) show that the events $\{r_i(t + \tau) \in (S - b,S)\,|\,r_i(t) = S\}$ and $\{b^\tau(t) \leq b\,|\,r_i(t) = S\}$ are equivalent. An identical argument applies conditions on other initial prices. This allows the conditioning to be removed so that

(A4) $$\lim_{\tau\to\infty} P\{b^\tau(t) \leq b\} = b/D,$$

as claimed.

Q.E.D.

Lemma 1 demonstrates that for $0 \leq b \leq D$, $F^m(b) \to b/D$ in distribution. Application of Proposition 8.12 of Breiman [1968] allows us to take limiting expectations using the uniform density:

(A5) $$\lim_{\tau\to\infty} \{E[b^\tau(t)(D - b^\tau(t))]\} = \frac{1}{D}\int_0^D b(D - b)\, db$$

$$= \frac{1}{D}\left[\frac{Db^2}{2} - \frac{b^3}{3}\right]_0^D = \frac{D^2}{6},$$

as claimed.

To establish part (b), it must be shown that for τ sufficiently small,

$$(A6) \qquad 1 \geq \frac{E[b^{\tau}(t)(D - b^{\tau}(t))]}{DE(\Pi^{\tau}(t))} \geq 1 - \epsilon$$

for any given $\epsilon \in (0,1)$. To confirm this, pick a time interval τ below $\epsilon(D/g_H)$, so that the maximal inflation rate in any given period is below ϵD. Then,

$$(A7) \quad E[b^{\tau}(t)(D - b^{\tau}(t))] = E[\Pi^{\tau}(t)(D - b^{\tau}(t))] < DE(\Pi^{\tau}(t)).$$

In addition,

$$(A8) \quad E[b^{\tau}(t)(D - b^{\tau}(t))] \geq E[\Pi^{\tau}(t)(D - \epsilon D)] = (1 - \epsilon)DE[\Pi^{\tau}(t)].$$

Together, (A7) and (A8) establish part (b).

Q.E.D.

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